Turbulent conic lab vortices scale to wide ranges of violent natural actions

2. 3D GYRE FORCES CONTROL TORNADOES, SUPERCELL MESOSPHERES, CYCLONE EYEWALLS

By FRED E. HOWARD, JR.*

Retired, US Air Force Armament Laboratory, Eglin Air Force Base, FL, USA

SUMMARY

Single vortices and symmetric pairs of both fully immersed, sphericly complete conic gyres and surface half-vortices in water are driven centrally by 30 and 20 degree cones to high turbulence for measuring single gyre forces and two gyre mutual forces in 3 dimensions at near to far zone separations on scales of gyre turbulence diameters. The central conic drives are similar to the central heat and Coriolis-effect engines driving tornados, hurricanes, and storm supercell vorticity. Suitably immersed, each directly driven vortex has the entire spheric, or separately hemispheric, conic spiral and toroidal flow anatomy of water particle velocity from the first paper of this series. In surface hemigyres, radial forces between a pair of vortices with initially vertical axes are measured by small angle deflections of calibrated motor-mount pendulums, which are about two meters in length. Farther off vertical, each single gyre shows an axial force, or point thrust (**PT**), like that of a descending tornado. **PT**-corrected attractive and repulsive radial forces to 200 grams are measured between pairs of base-driven or point-driven cones, in same hand or opposite hand rotations, at increments of surface-symmetric axial angles to 150 degrees between axes of surface (lower hemispheric) gyres. Right angle gear drive units are also used, with rotary collars at mid-pendulum, to obtain **PT**/crawl-corrected radial/perpendicular forces to 300 grams, up to 180 degrees relative axial angles, on symmetric horizontally driven conic pairs of immersed, complete spherically developed conic vortices. Use of two 2-to-1 gears enables optically measured angular drive rates at 16:1 gear ratios of X/4, X/2, 1X, 2X, and 4X (within motor loading limits.) At 2X gear ratio, water viscosity is varied by factors of 1/2 and 2 (4:1 ratio) with small density changes. Drive power and vortex energy content are recorded. More limited trials sample asymmetries of relative axial angles and cone size, also surface roughness. Force/energy equations are derived for the symmetric experimental conditions employed. Weather vortices correlate with these lab vortex forces/anatomies, e.g.: Conic **PT** can push tornado funnel hemigyres to the surface from mid-level cyclonic flows, against up-drafts. In clear mesosphere, spheric gyre spiral waves may couple supercell vorticity to airglow gravity waves and pump ionic air down for sprites/blue jets. Fast moving hurricane eyewalls are distorted by offset of mesopheric downwash in the upper hemisphere. Gyre forces reverse unstably with decreasing separation of gyre pairs. Field & lab sequels are implied for science, forecasts, and safety of air/land/sea navigation.

*Mail address: 306 Gardner Dr. NE, Ft. Walton Beach, FL 32548, USA; e-mail: fredhoward1@cox.net

Keywords: Conic driven vortices Surface vortex Immersed vortex Tornadoes Mesospheric gravity waves Mesosphere sprites Storm supercells Turbulent vortex pair forces Hemivortex spiral waves Toroidal gyres Global vortex force angles Spherical vortices Single vortex forces Mesocyclones Turbulent boundary layers Cyclones Hurricane eyewalls Open eyewalls Downbursts Microbursts Blue jets Night airglow waves

1. INTRODUCTION

(a) Objective, general background, and systematic approach

This second note of a series provides the current data in a new approach to empirical vortex forces and their correlations with many energetic weather effects from the surface to the mesosphere. These research notes originated from lab experiments in water with conic vortices which are axially driven by solid cones to sufficiently high levels of rotational turbulence that both the anatomy of the velocities of water particles in the primary and secondary flows of single turbulent gyres (Howard note one, submitted for review), and also the forces developed by these turbulent single gyres and dual gyres (reported herein), could be readily observed and measured.

The experiments of this series of notes were initially intended as preliminary trials (in connection with other on-going research in electronic vortex currents) that might lead to a more precise and narrowly focused experiment elsewhere. However, the initial exploratory data show significant immediate correlations with mechanical force interactions of the strongest types of weather vortices, and add at once to the scant empirical data on forces involving secondary flows in turbulent conic gyres. The rarity of such data stands in contrast with the large body of literature on vortex experiments that are not as directly relatable to violent weather, such as laminar surface gyres, two dimensional gyres without secondary flows, isolated cylindric or toroidal gyres, etc.

Vigorous weather vortices are generally conicly tapered, short in axial length, turbulent in flow structures (often with spiral waves), and concentricly driven by distributed heat release engines in a viscous medium under surrounding external pressures with Coriolis rotational effects of prior and added momentum. These factors energize the typical conic rotating form of net lower level flow inward, upward, and outward again in the cloud tops, with generalized signs of an opposite upper level subsidence or down flow in clear atmosphere above the clouds (e.g., Fig. 3, van Delden, 2003; and Fig. 8, Hoskins et al., 1985). Such overall weather anatomy of mechanical flow is most similar to the turbulent laboratory vortices of the first note on the 3D water particle velocities of centrally driven, spherically complete, immersed conic gyres with secondary (and tertiary) flow structures in distinctively different upper and lower hemispheres, as well as of separably matching lower hemispheric surface gyres (Figs. 3 and 2, Howard 2006) in which the base of the drive cone is submerged to about one diameter in depth.

The basic force experiments herein (Fig. 1b, simplified from Fig. 2, Howard 2006) are driven by the same 30 degree (whole angle), right circular cones and motors. The velocity-tracking particles and video camera of the first paper are replaced by twin force-measuring pendulums, which mount pairs of cones and drive motors. As before, this provides, within the primary flow body of each gyre, a set of known angular and peripheral velocities on a known and simple conic boundary layer surface of viscous rotational flow, on which forces can be measured in summed resultant. In the majority of these force experiments, two identical drive cones and symmetricly balanced variables of driving the gyres were chosen for most direct analysis of vortex forces over a wide range of variables, including one variation of the cone size and angle.

The principal emphasis of this note is first on derivation of empirical equations for measured vortex forces and their scaling characteristics. These equations and data then enable quantitative scaling of large mechanical effects in violent weather formations which are not well understood, and also define further field and lab research tests for improved forecasts of weather damage, and safety of air and sea navigation. (This systematics is directly extendable to tornadoes/waterspouts in vortex pairs with storm cells or rain bands of tropical cyclones, to unstable eyes of such cyclones, to destructive downbursts in open eyewalls, to microbursts near airports or sealanes, and to clear air turbulence and mesospheric sprites near storm supercells.)

(b) Outline of this report

The force-measuring pendulums are described in section 2. Section 3 shows the origin in hemisphere flows of conic vortex forces, as scaled for two cone sizes in deriving an equation for the readily observed gyre diameter (*GD*) of high centrifugal turbulence in spiral wave planforms. In section 4, wattages of the gyre driving motors yield estimates of energy stored in and transmitted by *GD* scaled volumes of rotating vortical fluids. Section 5 derives an equation for the point thrust (**PT)** force of a conic vortex and models **PT** data by *GD* scales. Sections 2-5 apply to all other data.

Section 6 correlates some force couplings of single experimental vortices with large scale weather. In subsection 6*(a)*, the axial **PT** force of the lower hemi-vortex in the lab is correlated in *GD* scale with that of a specific tornado funnel gyre in its descent to the earth's surface against the convective updraft of a large storm cell. In subsection 6*(b)*, outward acceleration by the centrifugal *GD* spiral wave disk of input fluid, which flows down to the disk through the upper hemisphere of a fully immersed lab vortex, is correlated: *(1)* With predicted continuous downflow of ionized (ionospheric) air above the cyclonic vorticity of storm supercells; *(2)* with the subsequent enabling above a specific supercell of mesospheric electric discharges called sprites, etc.; and *(3)* with predicted *GD* disk acceleration of the downflowing air spirally outward and up in the night air glow of centrifugal mesospheric gravity waves from the same supercell with associated forceful flight turbulence above the supercell.

In section 7 radial mutual forces in symmetricly driven pairs of surface and immersed vortices, are measured, corrected for single gyre forces, reduced to *GD* scaled equations, and interpreted in graphs for implications on predictably hazardous, unstable reversals of direction of mutual force between weather vortices. Section 8 has limited data on the three additional sets of mutual and single gyre forces which act perpendicularly to the radial mutual force, including forces toward or against coaxial alignment of gyre axes, with implications for initiation and irregular tracks of tornadoes. Section 9 has samples of force data for dual gyres in asymmetric geometries, and a joint correlation with sections 7 and 6b to irregularities of the eyewalls of tropical cyclones. Section 10 summarizes the principal conclusions, correlations of violent weather with lab data, and implications found, including needs for further lab and field research related to weather forecasting, damage potential, safety of flight, and surface navigation.

2. Description of adjustable, force-measuring pendulums and their use

The swinging axles of the two pendulums were 10 mm (3/8 inch) bolts, which were adjustably blocked in sliding positions of variable separation along an overhead cylindric bar for the sole bearing point of each axle. Each pendulum was made of two flat 5 cm metal bars with position scales. In the middle of the 2 meter length of each pendulum from axle to test item there was a freely adjustable sliding length and angle joint. At the foot of each pendulum there was a two-position, parallel or perpendicular mounting of direct drive to a 20 or 30 degree (whole angle) cone body by a reversible, 10 mm (3/8 inch) electric drill turning extendable 6 mm or 10 mm shafts. Resultant axial angles of each drive shaft, over a range of zero to 75 degrees from vertical, were measured to about 1 degree accuracy with gravity angle indicators before and after surface-penetrating runs for surface vortices. Voltages were checked and amperages were measured under the various loads, while optical rpm instruments were held near the chucks of the drills. (Power factors were not measured.) Pendulum deflections up to 3 degrees due to sums of mutual radial force $\mathbf{F_M}$ and any components of axial **PT** were observed along the horizontal radial reference line of separation between the centroids of volume (CV) of cones.

In most of the experiments with surface vortices, low friction guides along both sides of the pendulums eliminated significant lateral or twisting deflections. When lateral out-of-vertical-plane forces were also observed, any twisting force deflections were minimized in significance by the axle positioning structure, and their moments were not observed. The out-of-vertical-plane forces with surface gyres are the lateral mutual force $\mathbf{F_L}$ perpendicular to $\mathbf{F_M}$ and to the reference plane of symmetry of the cone axes, and a side "crawl" force which arises from each cone gyre whenever its axis is not perpendicular to the water surface, even when fully immersed. For immersed vortex experiments, pendulum changes (described next) rotate the reference plane of the two cone axes by 90 degrees around the horizontal line of $\mathbf{F_M}$ to the horizontal plane, disclosing the mutual force $\mathbf{F_P}$ which is perpendicular to both $\mathbf{F_M}$ and $\mathbf{F_L}$. The primary forces $\mathbf{F_M}$ and **PT** are each correctable for components of any other forces (the rest of which are discussed in section 8**.)**

For fully immersed gyres on horizontal axes, right angle gear trains were used below continuously adjustable rotary collars at mid-length of the pendulums. Combinations of 1-to-1 and 2-to-1 reversible gears, in sets of two for each drive train, provided drive gear ratios of X/4, X/2, 1X, 2X and 4X (times) the motor rotation rates under the various loads. The motor was overloaded at 4X gear ratio, with much reduced rotation rate and overheating.

On each readjustment pendulums were calibrated with weights and a low friction pulley for radial and lateral forces up to 240 grams force at 10 cm deflection, with spot calibrations to 400 grams, at an estimated average accuracy of 1.0 gram. However, the limit on measurement accuracy is that of estimating the average of constantly varying changes in the reading due to gyre turbulence. Five percent is rare. Ten percent is typical in midrange, but that is estimated to increase to fifteen percent or more at the extremes of the lower and highest cone rotation rates (with the least force or most intense turbulence.) When large attractions force cones or parts of pendulums into firm contact, the force required to pull them apart can be fifty percent or more above the plotted contact deflection data. (Adaptation of these spinning systems to the typical art of rigid strain-gauge force balances for static bodies was beyond the scope of these experiments.)

Section 3. Flow anatomy and equation for Gyre Diameter scaling reference

The principal feature of a strongly driven surface vortex is the central gyre diameter (*GD*) of apparently chaotic turbulence. The turbulence is actually organized usually in about six quasi-radial, almost standing waves (Fig. 1b, simplified from Fig. 2, Howard, 2006) which form a thick, subplanar disk of net centrifugal flow through complex rotations of the water parcels. The central inner ends of the spinning cores of the waves are conformally nested together in spirally trailing curves (sketched within the water of the entire wave bodies, not shown, between the wave cores.) Each spiral wave is itself a small vortex, fluctuating cyclicly with the energy of vortical wavelets around it in descending scale (not unlike many fractals.) The spiral wave arms of the main vortex subside from turbulence to smoothly spreading outward waves over a very short distance at the *GD* diameter. The diameter varies unevenly with each wave, while the whole planform rotates around the drive axis at a very low rate compared to the drive cone.

With individually trackable flow visualization particles, the *GD* disk appears not to change observably if the surface hemivortex, driven at a cone base depth of one base diameter (Fig. 1b), is further submerged, as a complete spheric vortex with viscous flow above the *GD* (Figs. 2 and 3, Howard 2006), and turned on its side with the drive axis horizontal. This stable *GD* disk is such a significant secondary part of the generation of gyre forces by transfer of momentum outward from the primary internal flows of the vortex along the surfaces of the drive cone (as summarized next) that the *GD* becomes the scaling reference for these forces.

There are distinctly different flow structures in the nominally upper and lower submerged hemispheres of the spherical flow anatomy around the *GD* disk (Figs. 2and 3, Howard 2006). The conic source of rotational energy is well within the lower hemisphere (Fig. 1b). The centrifugal primary flow off the flat base of the rapidly rotating drive cone couples viscously into the undersides of the outwardly expanding conelets of the secondary spiral wave arms, until each of these conelets is cylindricly accelerated to the maximum velocity and diameter of its individual internal core of cylindric rotation. This maximum occurs as the water parcels of the wave flow centrifugally outward in a corkscrew motion past the *GD* boundary (at about 2.56 times the diameter of the 30 degree drive cone with 1X direct drive.) At the *GD*, increasing losses of momentum to greater volumes of surrounding fluid reverse wave growth to a decline of size and velocities in each wave, so that its front turns in an increasing trail angle through 45 degrees from the local radius, and its turbulence is damped out in a short distance along the front.

The growth of the spiral waves on momentum from the centrifugal boundary layer of the drive cone base is also aided out to the *GD* by another, more massive, and almost laminar, primary flow, which is accelerated viscously up the sides of the drive cone to spiral outward centrifugally from the perimeter of the cone. With direct drive of the 30 degree cones, the line in vertical cross-section between the two interacting outflows off the base and the sides of the drive cone is 14 degrees above the plane of the cone base (Fig. 1b). Thus, there is an upwardly accelerated final velocity component along the 14 degree line from the primary inflow around the point of the cone. The downward momentum of base inflow (whether with water coming in from the sides above the spiral wave disk in surface gyres or axially in immersed gyres) is also reversed in its axial direction (Fig.3, Howard 2006) and accelerated in the broad cone of the turbulent spiral waves with an upward component above the 14 degree section line to as much as 30 degrees above the base plane. This also tends to maintain a fluid pressure head above the center of the disk and the cone base, while the greater rate of removal of fluid around the cone point tends to reduce the pressure below. In result, there is a net reaction thrust as with a rocket motor along the axis of the cone in the direction of the cone point. This reaction thrust is called point thrust (**PT)** herein, and is defined numericly in equation (2) of section 5.

In addition to transfers of disk momentum to tertiary conic eddies that appear to arise largely from the 45 degree bends of the spiral waves near the *GD*, there are toroidal flows that contribute to vortical force effects, with power data in section 4. Through the *GD* outward to about twice the *GD* radius (Fig. 3, Howard 2006), much of the spirally out-flowing momentum around the spiral wave disk is concentricly coupled to two large, dissimilar, secondary toroidal flow rings centered around the primary axis in their respective upper and lower (nominal) hemispheres. When undisturbed, these two toroids, sketched herein as outlines (Fig. 4b), act in part as momentum storage cells. On their toroidal flow returns toward the primary axis during their corkscrew particle movement around their rings, the two toroids then feed back some of the flow (and the contained eddies) axially into the primary flows along the cone surfaces. Part of the toroidal momentum is viscously transferred outward almost sphericly into the surrounding fluid, supplementing the unstored momentum transferred outward and upward by the spiral wave disk. In the upper hemisphere, in addition to pulsing variation of toroidal flow in the vicinity of each spiral wave, there is a more rapidly impulsive fluctuation of flow observed in and around the upper toroid as a vibratory motion of neutrally buoyant flow visualization particles driven by the turbulence of the spiral waves (Howard 2006). This vibratory component of momentum transfer outward propagates as directionally rotating or polarized pressure waves or sound-like waves which contribute to forces in the upper hemisphere. The particular flow structures which mediate specific force interactions between two conic vortices will be dependent on both the global relative attitudes (Fig. 4b) and the separations of the gyres on the *GD* scale, from near zone contact of the drive cones with flexible distortions of the flow structures to far zone separations with relatively low distortion of flow.

Whether the cone and the entire gyre are fully immersed, or the cone base is immersed one base diameter for a surface gyre, the *GD* dimension in centimeters (and the proportional thickness of the turbulent spiral planform) is primarily a function, equation (1), of the size and shape of the cone, its rotation rate, and the viscosity and density of the fluid. Accordingly, the plots versus cone peripheral velocity of *GD* data for surface and immersed gyres (Fig. 1a) fall on a single set of curves for different cone sizes. Two types of right circular cones were used in the experiments that establish this. One type of cone (with a set of four) was of maple wood, lightly sanded to remove most lathe marks on the sides, and not polished. These four cones were of 30 degrees nominal whole angle within about 1%, with 7.9375 ±0.05 cm base diameter, and were truncated at 12.38 cm length for a cap nut fastening of the 6mm axial drive shaft. The other type of cone (with a set of two) was of polished brass threaded in the base for a 10mm shaft, with 20 degrees whole angle, 3.175 ±0.01 cm base diameter, and 8.7 cm length to a slightly rounded tip. (Subsequent attempts to attach grit, or other roughnesses, to the polished brass, equivalent to the roughness of the wood, did not have an effect on the *GD*, though there was a detectable force effect noted later.) The respective base areas A_B were 49.48 cm^2 and 7.917 cm^2. The respective side areas A_S of the cones were 195.36 cm^2 and 43.87 cm^2. When directly driven (at 1X ratio) in city tap water at 20°C, the respective base peripheral or tangential velocities V_P were 872.9 cm s^{-1} and 356.5 cm s^{-1}. At that standard temperature the approximate fluid density ρ (from handbook tables for distilled water) is 0.998 gm cc^{-1}, and the viscosity η is 1.005 cp.

In the larger tub used for full immersion of gyres with horizontal axes, water temperature variations to 18.75 and 16.2°C with changes of nominal density up to 0.999 gm cc^{-1} and of viscosity up to 1.11 cp had no observable effect at 2 X drive gear ratio on *GD* or forces with their typical uncertainties of 6 to 10% (though the 2 X peripheral velocity of the 30 degree cone did change perceptibly by about 0.8%, shown in doubled velocity ticks on Fig. 1a and elsewhere.) Then water at 49°C (which was re-measured for every run) provided briefly a nominally halved viscosity of 0.56 cp with density of 0.989 gm cc^{-1}. A longer lasting change with 20% by weight solution of sucrose at 18°C doubled the nominal reference viscosity to 2.08 cp with a density of 1.085 gm cc^{-1}. At the 2X drive gear ratio, the 30 degree cone's *GD* and peripheral velocity did vary significantly (Fig. 1a) for these changes. (For ready tracking of such variations, there are below the peripheral velocity scale of the figure, separate tick mark scales and brief identifiers for the test conditions for each cone size at specific velocities.)

In the larger tub with two right angle drive gears (to provide horizontal axes and full immersion of the gyres) the frictional load of the double-geared train at 1X (2 gear) ratio reduced the average peripheral velocity V_P of the 30 degree cone for immersed gyres to 789.35 cm s^{-1} (in near standard water at 19°C.) This velocity [rather than the 1X direct-drive average velocity of 872.9 cm s^{-1} in surface gyres (with separate 1X tick marks on the scales for both cone sizes)] sets the reference velocity for comparisons with data (Fig 1a and elsewhere) obtained with the X/4, X/2, 2X, and 4X gear ratios, which use two gears in immersed gyres. [A number of horizontal runs at 1X (1gear) with a single right angle 1 to 1 gear and V_P of 832.6 cm s^{-1} obtained data only in mutual forces. There were also a few trial runs at 2X and X/2 (1 gear.)] Note that within the experimental uncertainty the dimensions of the *GD* disk (Fig. 1a herein and Figs. 1a to c, and 2, Howard 2006) were not observed to change with the velocity variations over the range of 1X conditions. The 4X series was run in near standard 17.5°C water, which was freshly refilled after removing traces of sucrose solution used at 2X. The motors were so heavily loaded by the 4X gear drive of the 30 degree wood cones that the peripheral or tangential velocity was only increased by about 50% (over the 1X gear ratio) to 1232.0 cm s^{-1}. The comparable 2X velocity was similarly limited to 1072.4 cm s^{-1}. The drive train with the two right angle gears also reduced the 20 degree brass cone's peripheral velocity at 1X to 328.2 cm s^{-1}, and limited the 2X case to 570.98 and the 4X to 977.77 cm s^{-1}. (With the smaller cone there was much less indication of overloading the motor.)

Within these ranges of variation of test conditions (which will also be cited for force measurements where applicable), the *GD* in centimeters varied (Fig. 1a) as:

$$GD = d_c \left[1 + e \left(\frac{\rho}{\eta} \right)^{0.6667} V^{A\sqrt{V}} \right] , \qquad (1)$$

where d_c is the cone base diameter in cm, $V = V_p/1000$ with the peripheral velocity of the base V_p measured in cm s^{-1}, the coefficient e is the base of natural logarithms, and $A = (2\,A_b + A_s)/100$ with the two component areas in cm^2. Note that V has its own square root in its exponent. Also in that exponent, the area of the base in cm^2 is given twice the weight of the area of the sides because of the higher turbulence and peak velocity in the spiral wave flow off the base which is the principal turbulent determinant of *GD*. [However, in scaling up usefully from the cone sizes in the experiment (for estimates on large weather gyres), the A in the exponent may take the *GD* calculation off scale, unless: (a.) The units of length measure for the base diameter of the hypothetical weather cone (and the *GD)* are chosen such that it is between 1 and 10 units, though they may be unusual units. (b.) The units of length to be squared for the areas of the sides and base of the weather scale cone are the same units. (c.) The peripheral velocity of the base of the weather cone remains in cm s^{-1} units.]

The last term of the equation (1), for the additional diameter of the *GD* above the diameter of the drive cone, contains elements of the Reynolds number, but employs those elements quite differently. This empirical usage describes a doubled variation in radius with a proportional increase in the thickness of the boundary layer attached to the base of the drive cone, as shown in the 30 degree slope over half the *GD* of Fig. 1b (Figs. l and 2, Howard 2006). This implies an empirical scaling relation to the thickness of turbulently accelerated boundary layers in this geometry. (Another cooperative relation appears in the next section.)

In Figure 1a the excessive deviation between the measured and computed values for the 30 degree wood cone in the 2X low viscosity (hot water) case is attributed to the very rapid evaporative and conductive cooling rate of the water, much increased by the local turbulence during measurement of the *GD*. (Accurate local temperature and viscosity requires a short time constant probe always within the *GD*.) The much higher percentage deviation on the 20 degree brass cone at 4X drive gear ratio was probably caused by an unexpectedly large effect from the 10 mm drive shaft in the small cone at this high rate. (A similar effect may be present in the 4X force data on the 20 degree cone.) Otherwise, the equation (1) approximates the limited data.

These data show clearly that at 2 times higher viscosity (and very small change in density) with the 20% by weight sucrose solution the *GD* (Fig.1a) is much smaller than it is with the tap water. Also, since there is less material mass in the smaller volume being driven to turbulence, and since that smaller mass is coupling momentum losses outward over a smaller area, the cone peripheral velocity is higher than it is with the lower viscosity of the tap water. In addition, it will be seen in the next section that the current drawn by the motor, the driving power absorbed, is significantly less with the higher viscosity fluid than with the tap water.

Exactly the reversed kinds of change from the normal condition with tap water occur, with both the *GD* and the cone rotational velocity, when the viscosity of the experimental fluid is reduced by approximately half in the heated water at ≤49°C (Fig. 1a) compared to the tap water. The *GD* and the spheric gyre volume it defines both expand, and the cone rotational velocity decreases due to increased loading. These effects are approximated, as graphed in the figure, by the numerical relations of the equation (1). Also, it will be seen in the next section that the drive current, the loading on the drive motor, increases distinctly with the reduction of viscosity by about half. This effect from expansion of the gyre volume and energy content (and dispersal) in inverse ratio to a 2/3 power of the viscosity is also substantiated by the mean trends and peaks of the repulsive mutual forces for these changes in viscosity (Figs. 7i, j, and k). The attractive forces are less definite and may indicate an opposing trend at small separations between gyres.

Section 4. ELECTRIC DRIVING POWER AND VORTEX ENERGY

Currents drawn by the electric drive motors for the 30 and 20 degree cones were measured for each significant change in the turbulent experimental conditions. Data were taken with both motors running, first dry with only air loads, and then wet under the experimental loads with gyres far apart and having little mutual force. Voltages were observed not to vary from the no-load value after the starting surge. The errors in these measurements are estimated at 2% to 3%, but the errors in subsequent power and energy estimates are not well resolved. The power consumed in maintaining turbulent gyres at constant size is apparently the difference in watts between the air load and wet load data. These differences plotted as gyre driving power (Fig. 2) under the various conditions of section 3 are probably upper limits. No positive correction was estimated for gear train losses at the higher angular rates with the air loads, and no negative correction was made for (probably larger) frictional loss in the drive train from the much increased loads of the gyres. Since these errors may be very large in the 4X drive gear ratio with the 30 degree cones, those data are only indicated well beyond the top of the figure.

A separate attempt was made to measure power difference due to mutual force between immersed gyres close together at 1X (1 gear) drive ratio with 30 degree cones. The best difference measurement obtained with the pendulums was something less than 6.6 watts due to maximum attractive force in the vicinity of 300 grams (compared to about 25 watts gyre drive power without mutual force, not plotted.) This power difference included unknown frictional losses due to unsteady contact between the two 30 degree drive cones. (Again, recorded rigid force balance instruments, beyond the scope of the experiments, would be much preferable.)

After the starting surge to bring the drive motor up to speed, average currents during the build-up of the turbulent gyre were estimated to be very similar to the running currents after development of the gyre to full size and turbulence. Averaged stopwatch estimates were then made of the length of time in which rotational energy is built up in the gyre while developing the gyre to full size. This provides an estimated approximation of the watt-seconds or joules of energy stored in the turbulent rotation of the individual gyre under various experimental conditions (Fig. 2). The data for 1X (direct) gear are from vertical surface gyres with the base at the standard depth of one base diameter, and all others are from fully immersed gyres. The two groups of data are generally consistent for 20 degree cones. However, with the larger gyres from 30 degree cones, the length of time required to establish adequate water inflow from the small hydrostatic pressure head above the centrifugal spiral disk in the surface gyre, can apparently be a significant delay that makes this 1X (direct) data point inconsistent (and invalid.)

The trends of the combined estimates and measurements (Fig. 2) are similar to the *GD* data curve (Fig. 1a). At 2X gear ratio, the variations of watts and joules with viscosity are like those of the *GD* curve in the negative slope of the crossing line with viscosity (due to the inverse relation between gyre volume and viscosity of the fluid. A similar effect on mutual force appears in later sections. Note again, also, that the low viscosity measurements in heated water were made at a lower than intended temperature, and the measurements are low.)

The rotational energy and angular momentum in the immersed gyre are stored in two volumes, of which one is the nearly spherical region of strong primary and secondary flow at about the *GD* diameter. The included smaller volume is that of the turbulence itself in the thick subplanar disk, or wide and shallow cone, of the spiral wave planform within the *GD* (Fig. 1b). Over the range of these experiments, this volume is estimated to have a turbulently variable thickness proportional to its diameter, so that both volumes scale with the cube of *GD.*

A convenient scaling value to compare proportional volumes of gyres in the 30 degree cone experiments with the power/energy data is the ratio of the computed diameter on the *GD* smoothing curve (Fig. 1a) to the diameter of the 30 degree cone at the centroid of the total surface area of the cone. This diameter d_{ccs} is approximately 6.3 cm. The cube of that variable ratio of the two diameters computed for the principal line of the *GD* curve (Fig. 1a) is plotted in a dotted line (Fig. 2) as a volume ratio for bodies of the variable *GD* diameter to similar bodies of the fixed d_{ccs} diameter. This computed volume ratio scales in a very close match with the power and energy data, showing that both data are indeed essentially proportional to gyre volumes.

This proportional matching of turbulent volume ratios and power/energy curves implies that there is a threshold of relative kinetic energy content per unit volume of fluid near a shear interface, beyond which threshold value the volume of turbulence expands until the threshold level is again reached and is just maintained in the relevant volume on average. Take the case of the immersed spheric conic gyres in near standard water at the 1X (2 gear) drive ratio for which there are full data (Fig. 1a). If the volume of storage in all high energy flow is that of the sphere of the observed 20.3 cm *GD* diameter, or 4.2×10^3 cc, with the turbulent disk just above the threshold of turbulence on average and the rest of the sphere just below the threshold on average, then the approximately measured energy content (Fig. 2) of about 12 joules implies a TBL threshold in similar geometry of order about 3×10^{-3} joules cc^{-1} or joules gm^{-1} in lab water. This implication may scale to other fully turbulent fluids with the 2/3 power of the density to viscosity (in cp) ratio as in equation (1) and in the correction factor of later equation (17).

Since the thickness of the *GD* turbulent disk in the scaled schematic images (Fig.1b) (Figs. 2 and 3, Howard 2006) applicable to the full range of 1X drives (Fig. 1a) varies from about 4 cm to about 6 cm with a mean of 5 cm, it contains about 3/8 of the volume and the energy of the nominal high energy sphere or about 4.5 joules of energy. The sphere stores about 12 joules, but transfers 16 joules/second (Fig. 2 watts) from the disk to the surrounding fluid.

Since the turbulent vibrational motion of the individual spiral waves, as noted in section 3, is clearly visible as repeated movement in the fluid above the disk, it is well below the 30 to 40 cps of drive shaft rotation, which the eye could not see as separate movements. Thus, these vibrations constitute a rotating field of acoustic pressure waves from the spiral disk as a tertiary effect within the ring of the base toroid. Assuming a non-linear refractive interaction with other gyres, the vibrations would add a component to net mutual forces. Such energetic effects would propagate in a laterally vibratory mode rather than at flow speeds in the medium. The effect would appear over a wide cone in the medium above the base of the drive cone (and in reaction below) and would die out in the far field as the usual inverse square of the radial distance of separation. The definite observation of this fluid vibration shows that a distinct fraction of the estimated 16 j s^{-1} of energy dissipated into the water medium from the entire sphere in this case should be measurable over a very low acoustic frequency band separable from motor drive noise in any future experiments. Such a measurement may correlate with components of microseisms from intense cyclones and of mesospheric waves above convective weather supercells, as in section 6.

Section 5. POINT THRUST (**PT**) FORCE DATA AND EQUATION

In section 3 it was pointed out that flow structures of each conic vortex contain two rocket-like ejections of fluid, with added momentum in the base direction, and a reactive point-directed axial thrust force, **PT**. Any component of this single gyre **PT** force in a dual gyre measurement must be subtracted from the measured force to obtain the interactive mutual force.

PT data from each gyre in these experiments are plotted (Fig. 3) against the peripheral velocities of the bases of the two types of drive cones for the various drive conditions. For 1X directly driven surface gyres at various axial angles relative to the vertical, each measurement of the horizontal component of **PT** was divided by the sine of the angle to find the full axial vector, and these data were averaged. Data from surface and immersed gyres are generally consistent, with midrange uncertainties well below 10%. The more erratic immersed X/2 data for the 30 degree cone resulted from trying a single 2-to-1 gear as well as a two-gear train. The **PT** datum at 2X drive gear ratio for the sucrose solution with high viscosity was not clearly differentiated from the mean of the two values for small temperature differences near the standard viscosity, as the previous *GD* and power data were. The hot measurement for low viscosity, on the other hand, definitely confirmed the mean negative slope of **PT** with viscosity.

The empirical equation (2) derived to express the data in general form defines:

$$\mathbf{PT} = \frac{\sin\theta}{10\,\pi}\left(\frac{\rho}{\eta}\right)^{0.25} V_p^{\,1.525}\,\frac{A_s}{9.807\times10^{2}}\,, \qquad (2)$$

where θ is the whole cone angle, ρ and η are fluid density in gm cc^{-1} and viscosity in cp respectively, V_p is the peripheral velocity of the cone base in cm s^{-1}, and A_s is the area of the sides of the cone in cm^2. While the area of the base is not included, the sine factor implies that the square root of the relative base area is effectively included, as a reduced weighting of the effect of the base area. The sine factor also describes the vanishing of axial **PT** force for the ideal cylindric or zero angle limit of the small angle cone. At the 180 degree whole cone angle in an immersed flat disk, the sine describes the zero axial force balance of the two sides of the disk. The number 9.807×10^{2} is the conversion factor from dynes force to grams force, which was more convenient in data measurements. Again, the empirical equation for turbulent vortices contains equivalents of elements of the Reynolds number with unusual exponents.

The equation effectively shows that for the 20 degree cone to reach any selected axial **PT** force exhibited by the 30 degree cone, the peripheral velocity of the smaller cone must increase over the peripheral velocity of the larger cone by a factor of the 1.525th root of the ratio of the larger to the smaller sine times the similar ratio of the side areas. To do that, the angular rate of the smaller cone would thus have to increase by more than the ratio of the cone diameters. At those relative conditions the boundary layer attached to the side surface of the larger cone may represent an arbitrarily simplified approximate model at that surface of the tangential velocity field around the sides of the smaller cone at the scaled up rotational rate, provided that the ratio of the side surfaces, or volumes, or diameters, etc., stays within some reasonable limit, such as that the diameter of the larger cone base does not exceed the *GD* of the smaller cone at the increased angular rate.

This **PT** modeling implication between the two experimental cones can be checked by data (Fig. 3). The 20 degree cone at the measured 977.77 cm s^{-1} peripheral velocity shows 17.55 gm **PT** from the equation (2) with the usual viscosity factor very close to 1.00. Solving for V_p by reversing the equation in the same condition shows that the 30 degree cone would have 17.56 gm **PT** at 285 cm s^{-1}. The product of the ratios of sines and side areas for the 20 and 30 degree cones (with the 30 degree values in the numerator) resolves to 6.511, with a 1.525th root of 3.4162. This times 285 cm s^{-1} gives 973.6 cm s^{-1} required velocity for 17.56 gm **PT** of the smaller cone. That velocity is low from the 977.77 cm s^{-1} value by 0.42%. Using measured rather than calculated **PT** for this check of the modeling rule should not change the result significantly. To check the stated range of modeling, the measured *GD* of the small cone at about 11 cm or the calculated *GD* at about 16 cm are not approached too closely by the base of the large cone at 7.9375 cm diameter. It can be noted (Fig. 1a) that the *GD*s for the two cones with matched **PT**s are very nearly equal; the fluid volumes will also be similar. Likewise, the smoothed and extended power and energy measures for the two cases (Fig. 2) are quite similar. Such modeling may be limited to cases in which: The smaller cone is of a lesser cone angle and could be fitted entirely within the volume of the larger cone, with the cone angle ratio slightly larger than the square root of the base diameter ratio; the fluid density to viscosity ratio is fixed.

To summarize for impact on the sections which follow, the conditions analyzed for single gyres in sections 3, 4, and 5 include: 30 degree (whole angle) drive cones and smaller 20 degree cones; both fully immersed (sphericly complete) vortices and defined surface hemivortices; a number of angular spin rates over a range of drive gear ratios of 16 to 1; and three fluid viscosities with a range of 4 to 1. The data in sections 3, 4, and 5 apply in the following sections to each trial vortex, whether surface or immersed, single or in a pair, in same or opposite sense rotation of a pair or in either hand alone, as well as over a range of separations of gyres, and over widely varied, relative axial angles within pairs of vortices. (The range of some variables is explored at only a few settings of others, in cross-sectional samples of the potential data field.)

Section 6. WEATHER CELL CORRELATIONS WITH FORCES OF SINGLE CONIC VORTICES: DESCENT OF TROPOSPHERIC TORNADOES. ASCENT OF MESOSPHERIC SPRITES.

(a) Vertical force on tornado vortices, two similar lab models

True experimental modeling of weather phenomena is essentially not feasible because of Coriolis effects, decreases of pressure and density with altitude, latent heat effects, temperature lapse rates, etc. Yet, given that forces in a tornadic thunderstorm cell have set an intense mechanical vorticity in motion in a gaseously fluid medium, some mechanical similarities to a laboratory lower, or pointed cone, hemigyre in water (Howard, 2006) can be observed. .

For instance, a series of very clear pictures with scale information have been published (Rasmussen *et al.*, 2001) showing a tornado below a cloud base at about 2 km altitude. The upper part of the funnel was measured photogrammetricly at 1.4 km wide. The dust and debris at the ground were about 400 meters wide. This would indicate a blunted cone of clear air rotation of about 28 degrees whole angle around the cloudy core of the lower funnel. It was stated that separately obtained doppler radar data at the time and location showed at about 4 km altitude a 43 m s^{-1} differential horizontal velocity from side to side in a tornado vortex circulation of about 1 km diameter. At the same time and map location there was also a 46 m s^{-1} differential velocity measure over a mesocyclone of 7 to 8 km diameter at some unstated altitude of not less than 4 km. The entire lower circulation was evidently conic, and the 7 to 8 km diameter may represent a large circulating volume of precipitation reflectivity near a *GD*-like structure above the funnel. The tip that touched the earth was clearly tornadic in reported ground damage effects over about the same path width as the photographed dust cloud.

In a published paper (Ziegler *et al*., 2001), from the same Project VORTEX, it is noted that a tornado at another time and place appeared at the surface after a mesocyclone at 2 to 5 km altitude above ground developed downward. This is similar to many eyewitness reports of the condensed funnels of tornadoes that appear to move downward from the clouds of intense storm cells. Since cyclonic circulations in general are associated with upward net flows (Rasmussen *et al*., 2000; and Markowski *et al.*, 2002), though downdrafts, horizontal vortices, and microbursts also occur in storm cell areas, the body of the intense vortical circulation of a tornado must typically propagate downward against an updraft around as well as within the body. If the body of circulation is taken as a real physical body (very much in the same sense as the earlier modeling interchanges), it would be consistent with the finding of **PT** in lab data to consider that a tornado body developed aloft may be effectively thrust downward against the convection upflow by pressure reactions to the ejection of accelerated air upward as a usual mechanical event within a rapidly rotating conic body in a fluid medium, and equation (2) applies.

It is noted first that the flow in the lower sides of the hemigyre of the lab case was almost laminar (Howard 2006). This is similar to the typical smooth appearance of the sides of the cited funnel image (Rasmussen *et al*., 2001) of a medium-sized tornado above the surface dust cloud. If the 43 m s^{-1} differential velocity measurement by doppler radar at about 1 km diameter is arbitrarily assigned to the photogrammetric measure of 1.4 km diameter as a 21.5 m s^{-1} rotary flow, and it is assumed that the angular momentum is conserved and maintained in expanded or compressed air parcels rather than being significantly coupled away to surrounding fluid over the flow period, then at the 0.705 km diameter base of a 20 degree cone (nearer the core of the tornado), and 2 km above ground at sea level (rather than the unknown ground level in Kansas), moist summer air particles of the same angular momentum would have a rotary velocity of 43 m

s^{-1}. (Air closer in at the 400 m diameter core would similarly project conserved angular momentum at 75 m s^{-1}, which is high enough to do tornadic damage of the type observed.) This estimated conservation of angular momentum is roughly consistent with the empirical scaling ratios described earlier in the lab data (outside the rigid body of the drive cone.)

Since data are available only on the static case after the tornado has already descended to the ground, it can only be assumed that whatever balance of forces exists in this static case would be similar to the balance further aloft if the tornado tip were just forcing its way down slowly through the cloud base. Assuming that the **PT** equation (2) would not change its form over such a scale increase as that considered here, and arbitrarily taking the core circulation as a 20 degree conic body with a base peripheral velocity of 43 m s^{-1} at 2 km above the tip of the cone at sea level in moist summer air, this would yield a total estimated **PT** force of 1.2394×10^{10} gm available to push downward on a tornado body of that scale. Assuming a flat plate drag coefficient of 2 for a worst case of flow interferences in the usual drag equation, over a circular projected frontal area in moist summer air at about 2 km above sea level, the tornado body might face an upward drag force of 3.5894×10^4 gm m^{-2} of frontal area from an updraft flow of 20 m s^{-1}. Such a drag force might strip away the toroidal flows, and the GD disk also might not be fully maintained, but might be bent upward in a narrower exhaust cone. Assuming that any reaction thrust efficiency increases and losses would balance within a minor effect, the **PT** force estimate expressed on the putative cone base diameter of 0.705 km diameter would give the tornado funnel a downward thrust of $10.l9685\times10^4$ gm m^{-2} of projected area. Thrust of that order of magnitude would be suitable for bringing such a tornado cone down to the earth's surface against the estimated drag. In this scaled estimate the velocity of the updraft surrounding the tornado body at the base of the storm cloud could increase to 33.7 m s^{-1} before it would balance the down thrust of the tornado and force it to retreat aloft.

Another feature of the laboratory gyre demonstrates **PT** in a conic vortex more like that of the tornado in existing without the intrusion of a solid central driving cone. Large primary vortices often generate locally intense secondary vortices, such as the conic tip of each spiral wave (Fig. 1b herein and Fig. 1b-c, Howard 2006). These small conic gyres have a frictional or viscous exterior drive from the base of the primary drive cone, which prevents the usual toroidal flow around the sides. The core of each spiral tip arises near the axis of the primary gyre on the base of the drive cone through amalgamation of tiny initial whorls on the boundary layer of the drive cone, similarly to the coalescence of small vortices noted in the literature (e.g., Riccardi and Piva, 1998; Smyth and Peltier, 1994; and Melander *et al.*, 1988). The water particles within each of those secondary gyres move outward through the spiral wave planform in corkscrew spirals under their internal rotary flow combined with the primary centrifugal flow (Fig. 1b). The pointed inner end of the conic tip of each gyre itself vibrates in and out in a limited motion, but does not move centrifugally away from the vicinity of the primary axis. The fluid particles in the rotary core body of the small conic tip must necessarily experience an effective **PT** reaction that holds the body inward toward the drive axis and forward on its own axis against the drag of the overall centrifugal flow. As with the estimated **PT** forces in tornadic hemigyres before and after they come down against updrafts, these little secondary conic gyre bodies can only be in balance between the two opposed forces.

The spiral wave tips do not have the axial symmetry of the main gyres from which equation (2) was derived. The conic tips or conelets are not right circular cones; their axes and bodies are bent. On each of the two quarters of its sides adjacent to the bottom driving quarter, the flow of each conelet is in dragging shear along its areas of flow contact with the opposed adjacent rotary flows of its two neighbor conelets. On the top quarter of its sides, each conelet is viscously losing rotational momentum at a lower rate to the upper inflow of fluid which is not yet accelerated outward. Still, approximate estimates of the main balance of forces can be made.

The conelets of the spiral wave cores (Fig. 1b herein and Fig. 1b-c, Howard 2006), are sketched as about 20 degree cones for schematic clarity to the eye with about 60 degree separations in the most typical case between centers of these small secondary conelets. But as noted earlier, the full flow bodies of the spiral arm waves fill the areas between the cores with outer circulations, so that the full wave is much larger. Consequently, a very small 30 degree conelet of length equal to the 3.96875 cm radius of the main 30 degree wood drive cone could be a suitable estimate model. The axis at the centroid of volume and water mass of that model conelet would be approximately horizontal above the perimeter of the main 30 degree wooden drive body at about a 45 degree local angle to the tangent. Per equation (2), in lab water with the usual density/viscosity ratio very close to 1, and rotating at the peripheral velocity of the maximum core rotational velocity of 229 cm s^{-1} actually measured in spiral wave cores and shown in the referenced figure, the model conelet would generate a **PT** of 867.23 dynes force. Even in the absence of toroids and possibly of full *GD* due to interferences, this **PT** would be expressed over the conelet body's effective flat base of 1.0634 cm radius. The 45 degree component of this **PT** along the centrifugal radius from the axis of the primary large wooden cone resolves at a centripetal **PT** force component of 172.6 dynes cm^{-2} in the base area of the conelet body and, thus, on each cubic cm of water with its one gram of mass just within the body. One of these 1 cm cubes will just contain the CV and center of mass of the small conelet body and is arbitrarily taken to be located at the center of mass directly over the perimeter of the large drive cone at the stated 3.96875 cm radius from the main drive axis. (Note that the reference states that the higher measured flow velocities, such as the 229 cm s^{-1}, are lower limits of actual velocities due to geometric effects, and that the real conelet rotating velocities are probably 50% higher than the number used in this **PT** calculation; there is margin for rough estimation.)

It must be noted at this point that the entire spiral wave planform disk is not attached to the base of the 30 degree wooden cone which viscously drives the primary flow of the main vortex. The thick disk (Howard 2006) is only held in that viscous contact by its own weight, if the axis is vertical, and by the general external pressure working against the Bernoulli drop in vertical pressure (Prandtl and Tietjens, 1957) due to the higher velocity flow between the disk and the base of the drive cone. The spiral wave disk does not rotate with the angular rate of the base of the drive cone, but at only about 1 to 2 revolutions per second or less in the baseline conditions shown in the reference figures (Howard 2006). Without further estimate of flow effects, if the centrifugal reaction force on 1 gram of mass at the stated water cube location is calculated at a typical 1 rps with a velocity of 24.9364 cm s^{-1}, the apparent centrifugal force along the radius of rotation is 156.68 dynes. This would estimate a static balance of these two principal forces within 10%, which would exhibit the presence of a body stabilizing force which only the **PT** is likely to provide within the flowing water of the typical spiral wave in such conic vortices. For the grossly static positions of vibrating conelet bodies actually observed, the body of each conelet must move in or out radially in the spiral wave disk and speed up or slow down in the disk angular movement near 1 rps until its centrifugal and **PT** forces are in balance.

These estimates from empirical observations show that, when parts of a large storm cell are driven to sufficiently energetic rotation, **PT** force may indeed bring tornadoes down to the surface of the earth against the updrafts of air into the thunderstorm circulation.

(b) Vortex flow fields and force anomalies in mesospheric nightglow/sprite discharges

In contrast with the previously cited reports on tornadoes from Project VORTEX, which always discuss mesocyclonic vorticity of large storm cells, and in spite of the general association of vorticity with any large vertical convection in mid latitudes (e.g. van Delden, 2003; and Hoskins, *et al.*, 1985), there is very little consideration of vortical effects in the literature on the study of sprites and related electrical discharges which occur in the transparent air of the high mesosphere above thunderstorm supercells, especially in the western plains area of North America (and at other favorable locations around the world.) The emphasis has been on the widely reported vertical movements in and above these storms and the associated cloud-to-ground lightning at lower levels (e.g., the historical survey of Lyons and Armstrong, 2004, and their references). Probably that is due to the primary optical observations, in which many very significant convective weather cells do not exhibit much vorticity in their surrounding shells of visible clouds. One monograph synopsis of an extended (40 year) study of the anatomy of thunderstorm cells discussed mesocyclonic rotation of the cells in only two cases (Fujita, 1992).

Despite the lack of attention, some occasional evidence can be found of vorticity in conditions favorable for sprites. Recent daytime pictures from the STEPS program (Lyons and Armstrong, 2004) show a recognized wave cloud (that is spirally shaped) above the turbulent structure at the top of a large storm cell. Also, there is a radar image of another large CN cell that distinctly suggests vorticity at the time of producing sprite discharges (Lyons, Nelson, and Fossum, 2000). As that reference states, part of the data problem is due to the fact that not all large thunderstorm cells produce sprites, even in areas where sprites are frequently observed.

Fortunately, at least one very large supercell which did produce sprites, as well as glowing gravity waves in the mesosphere over the western plains of the US, was photographed in infrared by a GOES satellite twice in the same hours of the night as the sprite observations. These IR images show in the cloud tops a distinct, horizontal spiral wave structure (Fig. 2, Sentman *et al.*, 2003), which could be generated only by definite vorticity in the storm cell below it. Much more striking is the back-up video (made available by the senior author) showing from a distant mountain top the circular sectors of radially moving gravity waves of night airglow around the central sprite discharges above the supercell. At about 500 km distance from the storm the video frame interval (30 seconds) permits clear visual perception of the outward movement at about 85 m s^{-1} of the gravity waves as enormous ripples about 20 to 30 km from trough to peak and 40 to 50 km in wave length. One excellent video frame showing this is archived in the report. The report cites other references for these waves as arising from tuned organ-like pulsations of convection in the central storm cell with about the observed 10 minute wave period. That kind of pulsing may occur from turbulent couplings of adjacent upflows and downflows in a CN cell. Pulsing overshoots of convection surges have been photographed in elevation (Fujita, 1992; and Fujita and Arnold, 1963) rising a kilometer or more above moderate-sized anvil clouds (not supercells) at irregular intervals and subsiding typically within 10 minutes. (This is reminiscent of the microseisms in the earth from the eyes of hurricanes, which were much discussed at weather meetings before the era of satellites.) However, the short vertical length of the possible pulsing pipe compared to its expected diameter in such widespread storm cells as can cover most of the north-south dimension of Nebraska in the GOES images might not be expected in the presence of storm turbulence in the pipe to yield such well tuned regularity of wave emissions as are clearly recorded and reported. Since there was significant vorticity present, there might be another contribution from its spiral waves (as seen in the GOES image of this supercell) to the regularity of emission of circular gravity waves. The circular type of wave is always present in complete, spherical conic vortices going out radially from the spiral periphery at about twice the diameter of the turbulent *GD* (Fig. 1b herein and Fig. 1c, Howard 2006). Within the *GD* in the lab the centrifugal spiral flow is not only outward, but also upward at between 14 and 30 degrees. This would correlate in the gravity wave origin with a possible transparent stratospheric spiral wave disk which would couple momentum outward and upward into mesospheric gravity waves. In this flow structure the laterally spreading anvil cloud tops of CN cells of all sizes would indicate lateral flow below the spiral disk into the upper portion of a toroidal flow structure in the lower hemisphere of a spheric vortex driven by the CN. (Specially designed balloon-borne windsondes might be required to test these correlations.)

Though it may require repeated playback of the video (Sentman et al., 2003) to observe at the shallow angle of view provided by the combination of the distance, the earth curvature, and the 85 km altitude of the origin of the gravity waves above the storm, there is around that origin in the clear, but patchily glowing air, a repeatedly visible elliptical pattern of wave-like patches of opposite lateral directions of flow. This is an apparent counterflow such as would be projected laterally by a locally horizontal circle of nightglow circulation. This patchy apparent circle is in the same general region as the distribution of the sprites below the circle. (The 5 to 10 millisecond sprite discharges glow much more brightly than the faintly continuous night airglow. The sprites appear in single frames from time to time during the two hours of activity recorded with a frame of 25-second exposure every 30 seconds.) The circulation appears to be cyclonic and centrifugally distributed out from the top of the CN convection cell by its vorticity. It has a regular spiral wave pattern (with an upward component) like that of the lab gyres, which would contribute both regularity of gravity wave distribution and the radial momentum of the waves when beyond a central *GD* planform.

Since the tops of the clouds of such weather supercells can be as high as 20 km or more (Sentman *et al*., 2003), these spiral wave structures, with apparent *GD*s of as much as 150 km or more in the cited satellite image, would generate circulating effects by viscous coupling to a considerable altitude in the low viscosity air (sections 3 and 4) of the stratosphere and mesosphere. This could centrifugally pump outward the *GD*-level air very much like the upper hemisphere of the complete spheric conic vortices. By correlation of similarity to Figure 3 of the prior note (Howard 2006) the rotating downflow which would result in the center of the planform might reach the altitude of the ionosphere. This could bring previously ionized gases into the mesosphere for some reduced lifetime at the higher density. Since the down flow would (through Coriolis effects) generate anticyclonic pseudo-forces, the flow structure could be expected to change somewhat from that in the lab, but the centrifugal, laterally outward removal of air mass packets at the lower level should continue to pump ionized gases down (in the same manner as any centrifugal water pump.) The presence of ions and electrons, even at a much reduced percentage by increase of recombination rates due to density, would facilitate the electrical discharges seen in sprites, blue jets, elves, etc. Some level of ionization must remain in a column through the mesosphere since the discharges are largely seen in the areas that would be influenced by such downflow above the clouds, and gravity waves also glow in the outflow. These effects could be seen as evidence of vortical coupling in the upper hemisphere of these storms as sphericly active, fully immersed conic vortices mechanically similar to those in the lab.

However, when the anvil cloud at the top of the storm cell is displaced laterally from the upper part of the cell, the sprites also appear displaced over the anvil structure (Sentman *et al*., 2003; and Lyons, Nelson, and Fossum, 2000). In the previous discussion it was pointed out that the force balance which keeps the spiral wave planform close to the top of the cone base in the lab is dependent on the Bernoulli reduction of vertical pressure due to the higher rotating velocity in the water immediately adjacent to the continuing drive by the cone below. When the storm cloud matures and its vertical flow and vortical drive force coupling begin to decline, the momentum in the spiral wave structure will keep it rotating as a body until its momentum is viscously dissipated outward (Howard 2006). In the lab water this delayed decay of momentum is quite definite. When the continuing spiral wave pattern is no longer well attached to the storm cell by the Bernoulli effect, the spiral disk may blow to one side in any wind that displaces the anvil cloud. This would carry any ion downflow laterally (possibly through a bent channel at an angle dependent on the displacement rate) as well as carrying to the same side the higher electrically charged clouds which would then generate outside the storm cell the direct high-cloud-to-ground lightning stroke that usually triggers the sprite display. (A transportable ionospheric height measuring instrumentation may be able to check the validity of this implication. Anomalous propagation of ham short wave radio might detect the effect at times.)

Thus, there are distinct mechanical correlations of weather vortices with lab experiments on single turbulent conic gyres. The first paper of this series (Howard 2006) also correlated flow structures between lab experiments and tropical cyclones. In the following sections, the forces between pairs of gyres at various relative axial angles have correlations with tornado vortices adjacent to convective supercells, or with the findings of Project VORTEX that horizontal vorticity along a cool boundary may contribute by axial tilting to the vertical vorticity of tornadoes (e.g., Rasmussen *et al*.,1994; and Ziegler *et al*., 2001), and other weather effects.

Section 7. **PT**-CORRECTED RADIAL MUTUAL FORCES BETWEEN PAIRS OF GYRES AT RELATIVE AXIAL ANGLES---LIMITATIONS, VARIABLES, DATA, AND DERIVED EQUATIONS

For a condensed exploratory survey of empirical mutual forces between pairs of turbulent conic gyres, most of this research phase is limited to symmetricly arranged pairs of identically generated vortices from either of two sizes of right circular drive cones, with the single exception of trying both same and opposite senses of rotation at the bases of each pair of cones with each combination of the other variables. Also, limits on ranges of separation between paired gyres that could be explored efficiently changed with many variables. In the empirical data on record of about 900 force measurements, there are at least nine independent variables other than symmetry, plus several near equivalents of two of them, and there are derived variables such as *GD* and **PT** which have major interactions. It was possible to explore only the combinations of variables which were found by experiment to have the most general significance that might clarify force interactions of possible interest and yield equations on them for numerical and functional estimates over a range of scales. In result, the data are limited cross-sectional samples of the variables in a potentially much larger, multidimensional data field.

The main data organizing variables are by surface (*S*+) and fully immersed (*S*−) gyres, for which similarities are systematized where possible by setting the depth of the base of the drive cone for typical surface gyres at the depth of one base diameter (as noted with additional details in sections 2 and 3.) Gyre pairs in relative contra-rotation handedness (*H*–), with no other difference in preset variables than the sign of *S*, are so similar in forces that they usually reduce to the same equations. Gyre pairs in same handedness of rotation (*H*+), on the other hand, are distinctly different in the scale of forces between surface and immersed cases, and the derived force equations are adapted for this. Same rotation *H*+ cases in *S*+ (surface) also require special treatment for separation of effects of symmetric pairs of cones with either 20 degree or 30 degree whole cone angles (θ), and for changes of force patterns with relative axial angles ($\pm\alpha$). Other *S* differences arise in a shift of critically distinctive force effects with *GD* scale of radial separation (*R*) between CVs of the drive cones of a pair of gyres, and this also varies with α. Referenced to parallel and co-directional axes of cones (reading from base direction) where α =0 (Fig. 4a), symmetric relative rotation of cone axes with points inward (PI), or opposed to some degree, yields a positive (+) relative angle α of twice the rotation of each axis in the plane of symmetry (Fig. 4b). Symmetric relative rotation of axes with bases inward (BI), or opposed, is negative (−) rotation and results in $-\alpha$ angles. (These factors are organized for quick reference in Table 1.)

These new variables, plus those of prior sections, control the principal force analyzed herein, i. e., the **PT**(and crawl)-corrected mutual force $\mathbf{F_M}$ on each gyre along the radius of separation between the CVs of pairs of cones. Positive (+) force repellantly increases separation between gyres, and negative (–) force attracts gyres into reduced separation on the pendulums. (Other forces are measured with immersed gyres, as noted in sections 2 and 8.)

Figures 4 c, for a sample of narrowly specific empirical data, and 4 d, for the calculated and normalized, general mean trend of all the empirical $\mathbf{F_M}$ data recorded, illustrate typical experimental and derived curves for $\mathbf{F_M}$ versus separation *R* in *GD* units of two conic gyres in either same hand rotation (upper curve) or counter rotation (lower curve) at 1X direct drive angular rates of the cones. In the data curves (Fig. 4 c) the 30 and 20 degree cones have entirely different scales in centimeters of separation to match the single *GD* scale for both sizes of cones with their different *GD*s at similar (within 10%) angular rotation rates. Also the vertical scale of force is a factor of 10 higher for the larger cones. (The later single-case graphs of this type are more readily comparable with separations plotted in centimeters and a tick mark for each GD.) An additional pair of curves (Fig. 4 c) for the 30 degree cones at +13 degrees relative angle α between axes, shows that there is not a great increment of change with the small variation from vertical of 6.5 degrees for each axis. It is important to give special attention to the unstable, or indecisive, cases of close approach (indicated in the figure by reversed arrows between two points) in which two vortices are observed to go abruptly from one semi-stable mutual separation and force measurement location to another, either between repeated runs, or once in a single trial run, or several sequential times per short run in a true oscillation. This instability is especially noticeable when there are abrupt reversals of the direction of force from repulsion to attraction, which is the most common type of case. There are also large swings away from a fixed position without going to another definite position. Multiple downward arrows at the points of least separation show that attractive forces on the cones have brought them (or parts of the pendulum) into contact (hit and stick together) with greater force than the pendulum positions indicate.

The forces may be characterized as essentially near field (NF) at less than 1.5 *GD* actual separation of CVs, or transitional field (TF) from 1.5 to 2 *GD*, and essentially approaching far field (FF) beyond 2 *GD* separation, though the transition effect does not effectively vanish until about 4 *GD*. Beyond that radius of separation, with more sensitive instrumentation, the mutual force would vary in the classic Newtonian inverse of radius squared. The force direction or sign for *H*+ same rotations of paired gyres usually reverses from repelling to attracting at small NF separations of 0.7 to 0.75 *GD* in equivalent consistency with other research on continuously varying (rather than abruptly changing) coalescence or merging of similarly rotating, non-turbulent (rather than turbulent) vortices by general flow analysis (e.g., Melander *et al*., 1988; Smyth and Peltier, 1994; Riccardi and Piva, 1998; and their references).

When the cones actually come in contact at minimum CV separation with either co- or contra-rotation, the force on each cone can be estimated from **PT** (Fig. 3) and the average fluid ejection angle of about 14 degrees or 0.25 radians (Fig. 1b). At an estimated fraction of 0.9 from interference between cones, there would be an empirical peak attractive force limit in the case with the 30° cones at the 1X direct drive V_P of 873 cm/s (Fig. 4 c) of about:

$$\lim_{R\to 0} |\mathbf{F_M}| = 0.9 \frac{|\mathbf{PT}|}{\tan 14^\circ} = 3.6 \ |\mathbf{PT}| = 340 \text{ gm}, \qquad (3)$$

which compares well with trends of measured data and computed curves (Figs. 8 and 10c).

However, in calculating trends for most combinations of variables, the radial separation *R* in the equations for these forces is not actually equal to the simple *GD* quantity as it appears in the sample figures. The real *R* must be multiplied by a variable combination of the *GD* and other variables to yield $f_R(R)$, which can be taken as 1 times the curve for comparison of Fig. 4d with the 30 degree cone data (Fig. 4c). (A calculated curve for the 20 degree case would have a quite different function.) The function compresses or expands the two calculated curves (Fig. 4d) horizontally (Figs. 6, 7, and 8) so that the zero crossing of the upper curve is effectively shifted to left or to right on the actual *GD* or cm reference scale to approximate the varying separations (with changes of other independent conditions) at which the radial mutual force direction in the empirical data does typically reverse from repulsion to an opposite attraction with reduced separation between two gyres of the same rotation. The lower curve (Fig. 4 d), which represents the usual continuingly attractive radial force (Fig. 4c) between two symmetric gyres in contra-rotation (in distinction here from asymmetrics, Fig. 10), is also compressed or expanded (Figs. 6, 7, and 8) with other variables to shift the knee of the calculated curve (not the data) horizontally to left or right on the actual *GD* or cm data scale, by separate rules (all as organized in Table 1.)

The cause for these *GD* shift (GDS) variations in the calculated curves can be seen in the miniature display (Fig. 4 b) of the outlines of the secondary formations of two complete spherical (immersed) gyres at +30 degrees relative axis angles. Whether immersed and spherically complete, or at the surface and reduced to hemigyres, two conic vortices present significantly different symmetric aspects to each other (at any given separation) as their axes are rotated relative to each other (Figs. 4a and b). The interactions and forces change by more than the trigonometric functions, with relative angle variation alone, as well as with other variables. But the basic *GD*s do not necessarily change with many of these variables. Consequently, the *R* function in *GD* units must vary in GDS with same or counter rotations. In addition, there must be changes of vertical force levels by means of additional coefficients for many variables, in which relative angle is again significant. There must, then, be two sets of variable factors or elements (for radial separation *R* and for relative axial angle α) in the single complete mutual force equation (4). [If more explanation is needed, an expanded back-up of this paper is to be posted on a reference website (Howard 2006 or the publisher's) at the time of this publication.]

The curves of Figure 4d are produced by subsequent equation (15a), which is the general mean of a variable coefficient in equation (4) controlling GDS. But that coefficient is used only with modifying factors, and it is one of two variable coefficients affecting radial mutual force in:

$$\mathbf{F_M} = \frac{1}{2}\,\frac{\rho}{\eta}\,\frac{A_s \sin\theta}{9.807\times 10^2}\;V_p\quad f(\alpha)_{\mathrm{TBD}}\;f(R)_{\mathrm{TBD}}, \qquad (4)$$

where the functional coefficients are chosen from the remaining equations (5 to 24) by the TBD (to be determined) rules summarized in Table 1, and the other variables and coefficients have been defined previously. While it is much simpler in the table and far more convenient in experimental set-ups to use degrees, the equations that follow must be understood to use radian angles unless otherwise specified. Here the sine factor is probably not valid more than 50% above or below the limited 20 to 30 degree range, and must often be adjusted by the functional coefficients even there. The remaining equations for the coefficients will be taken up in the logical order in which they appear from top to bottom in Table 1 (with one exception.)

In the general features of Table 1, since surface hemigyres do not have a complete upper hemisphere, they have a simpler equation organization with less than half as many entries in the table. The more complete and complex shape of the full spheric immersed gyre (Fig. 4b) has more variation of coefficients with angle changes. The table also organizes complex variations of the effects of all the other (only apparently simple) factors of equation (4), such as viscosity and peripheral velocity, and their interactive combinations. In the more turbulent regime of these experiments, the component factors of the conventional Reynolds number do not appear to have as simple and invariant a form as in more nearly laminar flows, nor the simple form that equation (4) taken alone appears to indicate. It is more quickly comprehendable and brief in description to discuss all the equation options by their general features before comparing them to empirical data in Figures 6, 7, and 8, with 44 subfigures. (The discussions of lab force measurements will then also include further correlations and implications to violent weather mechanisms.)

As indicated in Table 1, equation (4) for $\mathbf{F_M}$ applies in all cases of symmetric gyre pairs for which empirical data were gathered. So does equation (15a) which describes Figure 4d as the general form of $f_R(R)$ for the $f(R)$ factor in equation (4). However, equation (15a) is used only in the modified or adapted form of equation (15b) with its various selections of modifiers (C_i, C_j, and C_k) for most areas of use. In contrast, there is no single master equation for the $f(\alpha)$ factor in equation (4). Most of the adaptive modifications change the forms of the equations substantially, so that it is more suitable to substitute entire $f(\alpha)$ equations into equation (4) to adapt to the combinations of variables in the experiments. Table 1 takes up these $f(\alpha)$ factors first.

The simplest $f(\alpha)$ table entries are in contra-rotating vortices (with $H = -1$ where needed), which affect the non-reversing lower curve (Fig. 4d) and have separate column areas in each part of Table 1 for surface and immersed gyres,. The simplest columns are for 0 to +180 degrees α (Fig. 4a) with cone points inward PI and usually base-driven BD. (Obviously, electric motor drives of surface gyres through the surface can only go to about 150 degrees relative.) However, in some cases data were taken at 0 degrees with both BD-PI and point-driven PD-BI drives, and different 0 degree equation adaptations can be necessary to match some conditions with the different drives (though actually, there is no BI or PI relation at 0 degrees α.)

The $f(\alpha)_1$ equation (5) for positive α in that simplest area of Table 1 is complicated by approximating a departure of the empirical data from its apparent norm of 1 (one) for $f(\alpha)$ by a factor of about 1/2 at both π and $\pi/2$ relative axial angles (or +180 and +90 degrees.) The possibility of this kind of peculiarity was anticipated earlier in the discussion of Figure 4b. Accordingly, the equation (5) for Figure 5a involves a long and very simple series with disturbances of the sequence at the π and $\pi/2$ points, as:

$$f(\alpha)_1 = 1 - f_Q(y) = 1 - \frac{y}{10^9}\,[y-(1/6)]\;[y-(1/3)]\;[y-(2/3)]\;[y-(1)]\;[y-(4/3)]\;[y-(5/3)]\;[y-(2)]$$

$[y-(4)]$ $[y-(13/3)]$ $[y-(14/3)]$ $[y-(5)]$ $[y-(7)]$ $[y-(22/3)]$ $[y-(23/3)]$ $[y-(8)]$ $[y-(25/3)]$

$[y-(26/3)]$ $[y-(9)]$ $[y-(28/3)]$ ....$\{(1/2)-(H/2)\}$, (5)

where $y = |\alpha|/\pi/6$. It is not necessary to refine the series further at the present level of experimental uncertainty. (Using the zeros and picking the points off the curve of Fig. 5a is quite adequate.) The final multiplier on the series in brackets is needed only if it is preferable to have the entire equation revert to 1 for same rotations of gyres with $+H$, where it is not applicable, rather than following the table by removing the equation. (Similar vanishing factors or terms may be employed with the other equations of this note if desired.)

Somewhat similarly, equation (6) covers the adjacent areas of negative relative axis angle or $-\alpha$ from 0 to −180 degrees with bases inward BI (Fig. 4a) in contra-rotation of two gyres. (Note that the double entry in Table 1 for 0 angle indicates using the equation which fits the drive, point or base, especially with surface vortices.) Equation (6) has the effect (Fig. 5b) of inverting and reducing the variations with angle of equation (5) to approximate the typical trend of the empirical data. (Since the ripples of the curve between zeros are not used, they are only dotted in.) Accordingly:

$$f(\alpha)_2 = (3/4) + (1/2)\, f_Q\,(y)\ . \qquad (6)$$

However, in the area of 0 to −60 degrees α, point drive PD-BI/0, immersed, with contra-rotation, that coefficient is not sufficient and must be modified to:

$$f_{BI0}\,(\alpha) = 2\ f(\alpha)_2\ . \qquad (6a)$$

The variations of effects with same hand or co-rotations of two symmetrical gyres not only follow the reversing effects shown in the prior upper curves (Figs. 4c and d) with varying radial separation, but also are quite different in immersed vortices from those of surface vortices, as is indicated in Table 1 by the eight additional versions of $f(\alpha)$. The four functions which approximate the empirical data for immersed vortices in same rotation are based on equation (7) (Fig. 5c) with three adaptations for the effects of *Vp* in later equations (8, 9, and 10) for different zones of relative angle variation. The basic equation (7) is a very simple function of relative axial angle α between + and –π, as:

$$f_H(\alpha) = 1 - \frac{1}{17}\tan\frac{4}{9}\alpha \ . \qquad (7)$$

This equation (7) applies directly only to the range of + 150 to +180 degrees of relative axial angles in BD-PI same rotation of immersed gyres. For 0 degrees it must be adapted for *Vp* effects, as:

$$f_{HV0}(\alpha) = f_H(\alpha)\left[\frac{1}{2} + \left(\frac{1.5}{0.34}\right)(z - 1)\right] , \qquad (8)$$

where $z = (V_p/800)$ when V_p is not less than 750 cm s^{-1} for equation (8) only, but not for use of z in other equations. (Due to lack of empirical data, it is not certain how this equation should be modified at $\alpha = 0$ in immersed vortices for V_p less than 750. The uncollected term is useful.)

Similarly, the broad trend of the sparse data in this immersed same rotation area for α in the ranges +30 to +120 degrees and –30 to –120 degrees indicates that equation (9) is matched in V_p effects only for 90 degrees in either polarity, as:

$$f_{HV1}(\alpha) = f_H(\alpha)\frac{1}{\left(\frac{3z - 0.75}{2} + 1\right)} , \qquad (9)$$

where V_p in z is known to be applicable to all the experimental velocities tried in these conditions, but at 4X gear ratio there are no data.

However, with similar provisos, equation (9) may be slightly modified for the remainder of that range of α, 120 degrees and 30 to 60 degrees of either polarity, as:

$$f_{HV2}(\alpha) = f_H(\alpha)\frac{1}{\left(\frac{3z - 0.75}{6} + 1\right)} . \qquad (9b)$$

A similar wide range of V_p effects is covered for the −180 degree PD-BI relative axial angle for immersed gyres in same rotation by a more complex relation in:

$$f_{HV3}(\alpha) = 2^{f(w)}\, f_H(\alpha) \quad , \tag{10}$$

where

$$f(w) = \left(\frac{-\ 3\pi}{5\ \ z^z}\ w \ln w \right) \quad , \tag{10a}$$

where z continues to include V_p, and $w = 2z - (1/2)$. As previously indicated in considering the shape implications of gyres (Fig. 4b), exact coaxial alignments of axes may be associated with peculiarly discontinuous variations of mutual force, like the one described by the effect of these two coefficient equations on equation (4).

The complex factor adjusting for V_p effects in equation (10) is also applicable much later in the sequence of development of Table 1, at two otherwise non-similar PD-BI areas shown at the bottom of that table, as a special coefficient for modification of GD shift for V_p effects in equation (15b) as:

$$C_{VV} = 2^{f(w)} \; . \tag{10b}$$

Surface vortex pairs of the same rotation interact with the surface, when at positive PI symmetric relative axial angles off the vertical, in such a way as to produce between gyres an increased repulsion due to a raised plateau of water several cm high. (For more explanation, see the expanded back-up at the website for Howard, submitted.) This high peak in repellant mutual forces over a range of angles below π/2 in the PI +α co-rotating data for surface gyre pairs suggests an exponential impulse curve (the solid curve in Fig. 5d) and equation (11) with adaptation terms, such as:

$$f_A[\alpha] = 2 + \frac{3ye}{e^y} - 5 \sin\frac{\alpha}{9} + f_P(y) \quad , \tag{11}$$

where $y = |\alpha|/\pi/6$ (again) from $\alpha = 0$ to $+\pi$, and:

$$f_P(y) = \frac{y}{\pi \times 10^8}(y-1/2)(y-1)(y-3/2)(y-6)(y-7)(y-8)............(y-14) \quad . \qquad (11a)$$

These equations cover the broad trend of all the surface data for co-rotating positive PI angles as the general form of the trend and can be used in that way. However, these equations are not the best fit above 90 degrees, and also the data for the smaller 20 degree cone and the larger 30 degree cone at the surface are sufficiently bimodal that further adaptation in whole cone angle θ (Fig. 5e) to distinguish the two modes is preferable. Therefore (respectively), equation (14) is used from +120 (or possibly +105) to +150 degrees, and from 0 to +90 (or possibly +105) degrees equation (12) is used containing equation (11) as:

$$f_{A\theta}[\alpha]_{PI} = f_A[\alpha] \; f_{BM}(\theta) \, , \qquad (12)$$

where:

$$f_{BM}(\theta) = 1 - 0.35\cos\left[2\alpha + \frac{100}{1+(0.2\alpha)^2} + 4\,\frac{\alpha + 7.5(30-\theta)}{1 + \dfrac{100}{\alpha + 1 + 15(30-\theta)}}\right] - 0.015(30-\theta) + 0.015(\theta - 20) \, , \qquad (12a)$$

where the use of radian angles would be very inconvenient, and degree angles are employed. (But it is not known how far into larger and smaller cone angles such a phasing effect as Fig.5e should be extended by some unknown equation.)

For the BI negative α in surface gyres there is a much less impulsive increase above the normal level of the same rotation repulsive forces in the vicinity of –30 degrees (Figs.5d), and the following adaptation of equation (11) is used in that area:

$$f_B[\alpha] = 1 + \frac{2ye}{e^y} - 2.5\sin\frac{|\alpha|}{9} \, , \qquad (13)$$

where y is used again, as in equations (11 and 11a) and the absolute values of α apply. As shown in Table 1, equation (13) must be applied in the 0 to + 60 degree α region of immersed co-rotating gyres. This equation (13) is also used with adaptation for whole cone angle θ in equation (14) for the +120 (or possibly +105) to +150 degree sector of the positive PI angles of surface vortices, as shown in Table 1:

$$f_{B\theta}[\alpha]_{PI} = f_B\,[\alpha]\;\; f_{BM}(\theta)\;, \qquad (14)$$

which completes the demonstrated requirements for adaptation of the f(α) factor in equation (4).

The general equation for variation of effects (Fig. 4d) on mutual force from separation of pairs of gyres f(*R*) in equation (4) for both surface and immersed gyres, must account for the reversal of mutual force directions in co-rotation as *R* approaches zero. Therefore, the equation must be impulsive in nature for the same rotation portion of the dual curve. For this upper curve alone, consideration may be given to alternative similarly shaped functions (e.g., pages 143 and 231 of von Seggern, 1993), one of which might have the advantage of being adaptable to inclusion of the attractive force limit separately stated here in equation (3). However, the new equation (15) used herein describes both the upper (co-rotation) and lower (contra-rotation) curves (Fig. 4d) (not including the force limit), as:

$$f_R(R) = \left(\frac{1}{2} + \frac{H}{2} - \frac{16}{(1+3R)^2}\right)\;\left(\frac{1}{(1+R)^{R-1}}\right) \;+\; \left(\frac{H}{(1+R^2)}\right), \qquad (15a)$$

wherein *H* provides the appropriate change of sign of 1 for the upper or lower curve. This general coefficient equation for application to adapting equation (4) must also be computed with additional internal factors to modify *R* adaptively, as equation (15b) indicates:

$$f_R(R_{i,j,k,l}) \;=\; f_R(RC_i\; C_j\; C_k\; C_l)\;\;, \qquad (15b)$$

where the basic real *R* is read in the actual *GD* units for the measured data or for a selected real value before *GD* shift. $R_{i,j,k,l}$ is thus employed to determine the factoral value of $f_R(R)$ to be applied to equation (4), either by reading its factoral value roughly on the curve of Fig. 4d or calculating it in equation (15a). But the real *R* has an effect on equation (4) only after being shifted in *GD* value to $R_{i,j,k,l}$ by equation (15b). Here C_i , C_j , and C_k , and even C_l on occasion, are each equal either to 1 (one) or to any coefficient shown in Table 1 as applicable to the case in question. These coefficients are supplied by equations (16 to 24 and 10b) where the general indices for the coefficients above are converted to the appropriate specific indices. The coefficients then have the ultimate effect on equation (4) of either shrinking or stretching the scale of equation (15a) and Fig. 4d horizontally in *GD* units and thus of also shifting the affected one of the two computed curves (not the data) in the proper direction on the *GD* scale to fit the empirical effects of the many variables in each specific set of conditions. The two upper and lower curve shifts specified for any particular single relative angle case by the vertical columns in Table 1 will not usually compress or expand together or in the same way or degree. Thus, the co- and contra-rotating *GD* shift effects are independent of each other. These strong variations of mutual force with relative axis angle α and other effects are forecast from consideration of Figs. 4b and 1b with their clear indication of juxtapositional irregularities in mutual interaction forces with variation of relative axial angles.

The first of these adaptation coefficients for use in equation (15b) is an additional factor for V_p for all cases, as:

$$C_V = z^z \quad , \qquad (16)$$

where $z = V_p /800$ again. If $z = 1$, the curve is not shifted by this coefficient. If z is near one, the shift is small and may be negligible within empirical error. This coefficient is used (Table 1) without further factoral adaptation of equations (15a and b) only for same rotation surface vortices from –75 degrees to +45 degrees of relative axial angles.

For all contra-rotation cases, whether surface or immersed, the effective *R* from equation (15b) is multiplied by an additional factor for viscosity, which completes its adaptation for shift of *GD* scale of the lower curve (Fig. 4d) in surface counter rotation, but does not complete the adaptation for immersed counter rotation. This shift factor is:

$$C_\eta = (\rho/\eta)^{2/3} \quad . \qquad (17)$$

Each R in $f_R(R)$ of equation (15b) for the immersed counter rotating pairs of vortices (Table 1), must not only be multiplied by equation (17) for *GD* shift of the lower curve but also by an additional coefficient or factor, which is selected for a narrower range of α. For +180 degrees alone the two cone whole angles must be distinguished by a factor which is active on the 20 degree cones, as:

$$C_{2s} = (2 \sin\theta)^2 \quad . \tag{18}$$

Here it is not known how far this effect (and similar effects to follow) should extend into larger and smaller whole cone angles as a general relation.

For the broad middle range from – 150 to + 150 degrees α, the additional factor is active on the lower curve for the 30 degree cones, as:

$$C_{3s} = (3 \sin\theta)^{2/3} \quad . \tag{19}$$

For –180 degrees in contra-rotation of immersed gyres, that equation (19) must be inverted to adjust the lower curve for *GD* shift, as:

$$C_{3sl} = 1/ C_{3s} \quad , \tag{20}$$

which completes the list of dual factors used with equation (17) with impact on the lower curve (Fig. 4d) for contra-rotational gyres. However, this factor (20) is also used alone with equation (15b) for same rotation surface vortices from + 60 to +150 degrees, where it effectively shifts the zero crossing point of force reversal on the upper curve for the 30 degree cones.

For +180 degrees α in same rotation of immersed vortices, the reversal point shift is adapted for V_P (especially to distinguish 30 degree cones from 20 degree cones) by a change of the exponent to include the z velocity ratio in equation (20), as:

$$C_{3l1} = 1 /(3\sin\theta)^{3\log z} \quad . \tag{21}$$

For – 180 degrees α in same rotation of immersed gyres (with 30 degree cones), a further small change of that z exponent in equation (22) suffices. The same coefficient (22) is also needed from – 150 to + 150 degrees in same rotation of immersed gyres, but with the additional factor of equation (23) over that broader range of angles. Again in same rotation of surface vortices from –150 to –90 degrees, equation (22) will be needed but with the further factor assistance this time of equation (24). (Note that the 20 degree cones were not tested over the broad range of negative relative axial angles nor for as many positive angles.) Thus, the most widely applicable of the three coefficients remaining to be discussed is:

$$C_{3l3} = 1\,/(3\sin\theta)^{3\log 3z}, \qquad (22)$$

which is used by itself only for –180 degrees, but combines for broader coverage with:

$$C_{\mathrm{T}} = \frac{1}{1 - \dfrac{2}{3\,\pi^2}\tan^2 0.485\,\alpha}\;. \qquad (23)$$

Coefficient (22) also cofactors with coefficient (24), as noted, for the high negative axial angles of same rotation of surface vortices, as:

$$C_{\mathrm{SS}} = 5/(3\alpha)\;, \qquad (24)$$

where α is in radians.

Finally in Table 1, equation (10b) provides a special cofactor for *GD* shift in the cases of high negative axial angles α in contra-rotation of immersed gyres. This completes the demonstrated adaptations of f(*R*) in equation (4).

Several attempts were made to determine a general corrective coefficient for surface roughness of the drive cone. The 20 degree cones were of polished brass, to which thin coatings of roughness similar to that of the wood cones were glued. Finally, the polished surfaces were roughened to a level similar to that of the wood cones with the corner of a file and a hacksaw. Some of the force data did change, as shown in Figure 5f. In a few other cases the change, if any, was clearly less than the uncertainty in the very low level force measurements for the rough or smooth small 20 degree cones. The roughnesses did extend beyond the body diameters at most points on the sides of the cones, and the effective slight increase in side area may have been the sole cause of the changes. It is thus found that larger, polished and roughened metal, 30 degree scaled cones of identical peak diameters, with roughness only from small removals of surface (as by patterned chemical etching), would be required to obtain a definite result for roughness effects with this pendulum instrumentation.

To illustrate the general effectiveness of these empirical formulations (4 through 24) wherever definite experimental results were obtained, the subfigures of Figures 6, 7 and 8 show comparable sets of the measured and corrected radial mutual force $\mathbf{F_M}$ data and the computed $\mathbf{F_M}$ curves specified by Table 1 for each case. The data and the associated curves are shown in the reduced graphs of Figure 6 in juxtaposition with Figures 7 and 8 so that the eye can see at a glance the radical variations and general trends of the data. The noisy quality and the sparseness of the data for each narrow set of conditions are also easily seen, but the averaged broad trends of the data and of the equations that characterize the observed effects are quite clear.

The way in which surface vortices are limited in this series of research notes to having the drive cones nominally one base diameter below the surface does minimize the differences between surface and immersed vortices. However, the differences are significant in the high repulsive force peaks for surface gyres in the vicinity of 30 to 60 degrees relative axial angles, especially in the positive angles (Figs. 5d and 6c to f). In the experiments for this area the effect on turbulence in the *GD* was especially visible at the surface as the repulsive force increased, rather than decreased, (counter-intuitively) with separation of gyres for points inward PI with the 30 degree cones at 1X drive gear ratio over the NF 1 to 1.5 *GD* separation of CVs (Figs. 6c through f). These experimental conditions combine in the contact area between the gyres the amounts of opposed flow momentums that would otherwise be distributed over larger areas and be entrained in part into toroidal flow without such a well focused opposition of flows. [See the expanded back-up at the prior reference website (Howard 2006).] A similar repulsive force effect to a less striking degree is also present in immersed PI and BI cases (Figs. 7b, c, and 8e).

In some empirical cases the cones collide and stick together, indicating that the real attractive force is much greater than the pendulums can measure. The attractive forces (Figs. 8f, g, and i) appear at times to approach the limits predicted by equation (3).

A striking feature of the data (Figs. 6e and f, 7b, 8b and c, and 4c) is the frequent occurrence of ranges of unstable mutual radial forces in which, when CV separation is decreased along the upper curve (Fig. 4d) for same sense rotation under turbulent conditions, large abrupt changes in mutual radial and non-radial force may occur, including reversals from repulsive to attractive force and brief oscillations. These unstable effects appear both on the surface and in full immersion. They most obviously occur with same rotations, but contra-rotational cases of an equivalent abrupt change of the level of mutual force with oscillatory tendencies are also noticeable (Figs. 8d and i). The frequent abruptness of the empirical effect observed here in three dimensional conic flows with solid driving cores in the presence of strong secondary flow turbulences, is obviously related to many analytic studies (e.g., McWilliams, 1984; Melander *et al.*, 1988; Smyth and Peltier, 1994; Riccardi and Piva, 1998) of largely smooth mergers of laminar primary flows without secondary flow complications, usually in two dimensions with extensions to three dimensions. The present more extended empirical findings of the repeated presence of abrupt force changes and reversals at short separations of turbulent conic gyres on a *GD* scale should indicate distinctive consequences in natural events. This unstable abruptness must have correlative implications in weather and in oceanic water flow as a contributory source of some of the uncertainties in weather prediction, in navigation through ocean current eddies and flooded river outlets, and in flight conditions, as further noted in sections 8 and 9.

8. Non-radial or lateral forces of single and symmetricly paired conic vortices

Reports in the literature, such as those just cited on the merger of vortices, show the general effects of lateral forces (typically when two non-turbulent, two-dimensional gyres circle each other in the process of merging rather than approaching each other along the radius of separation), but, as before, without turbulent secondary flows and more complete 3D conditions.

The limited number of experimental runs of this section (and section 9) very briefly samples the comparative scale of other types of forces than the symmetric radial force of section 7 and the **PT** of section 5. This section is limited to forces perpendicular or lateral to the radial force in symmetric experiments. Additional limitations are due to the use of low friction guides on the pendulums to isolate radial mutual forces between surface gyres by eliminating lateral motion, including crawl. (In the immersed experiments it was necessary to determine a crawl force to correct radial and lateral forces.) Also, with pendulum instrumentation the number of definite measurements of lateral forces is limited by an uncertainty in the amount of force (perpendicular to the radial force) involved in maintaining coaxial alignment in the plan view of gyre pairs that begin an experimental run approximately in coaxial alignment. In other cases without initial coaxial alignment there is a definite force toward alignment, but the amount of that force is not readily isolated. Consequently, in Figures 9a and 9b (as well as in Fig. 10 in the next section) many data points for forces that are perpendicular to the radial force between CVs are limited to being plotted only at an arbitrary low negative value indicative of tending toward coaxial alignment of gyre axes or of holding them locked in coaxial positions, especially with contra-rotating pairs. In some cases the amount of force tending toward coaxial alignment and perpendicular to the radial force could be determined and is shown, for surface cases (Fig. 9b), and for immersed cases (Figs. 9a and 10). (The *GD*s, **PT**s, and V_ps for these force curves may be obtained from prior curves (Figs. 1a and 3) by using the gear ratios shown.)

The crawl force (Fig. 9c) corrected for in immersed experiments (Figs. 7, 8, 9a and b, and 10) is exhibited by a single conic gyre whenever its axis is not vertical to the fluid surface. The asymmetry of pressures and viscous reaction loads from adjacent fluid masses on the sides of the cone tends to make the typical tilted gyre crawl as if the downward side of the drive cone were getting limited viscous traction on a non-existent roadbed beneath it. When a base-driven, base-up, conic gyre is tilted to the observer's right from vertical and is in left-hand rotation about its own axis through the base, it tends to crawl away from the observer under the influence of a force which can be labeled positive in direction as increasing the distance of observation. (For the practical immersed gyres used herein, the effect was significant in the horizontal plane of the two gyre axes, and correctional crawl force was measured with the cone axis horizontal.)

At plus and minus 180 degrees relative axial angles, the symmetry of the perpendicular corrected forces around the radius of separation of the CVs indicates that $|\mathbf{F_L}| = |\mathbf{F_P}|$ and that their signs are the same, and thus both are effectively measured (Fig. 9a) for the sample conditions.

Figure 9a shows some typical mutual non-radial forces $\mathbf{F_P}$ at a wide range of gear ratios and CV separations in centimeters. For the very limited relative axial angular sample cases of only positive and negative 180 degrees, the lateral and in-plane non-radial mutual forces over a very limited 30 to 35 cm range of CV separation do not appear too inconsistent with the radial mutual force data (Figs. 7g, i, j, k, l, and 8g, i, j, k), nor with the adaptations of equation (4) for radial force in these conditions. However, both above and below that very narrow range of CV separations the differences from the radial forces are distinctive, especially for the co-rotational 2X gear ratio which indicated a considerable force toward coaxial alignment (negative direction) at greater separations for the only time a fully corrected force appeared to have this definite trend into negative signs with increasing separation in co-rotation with symmetric gyres. [This trend is not inconsistent with the inverted signs of data in asymmetric gyre situations (Fig. 10).]

With smaller CV separations than 30 cm, the range of positive data (Fig. 9a), for $\mathbf{F_P}$ repulsion of coaxial alignment of immersed co-rotational +180 degree α gyres at all drive gear ratios, is 50 to 100% higher than for the maximum positive radial $\mathbf{F_M}$ force data in the equivalent subfigures just cited. The radial force data for immersed gyres rarely reach such positive values consistently over a wide range of other conditions except at the small relative axial angles below 90 degrees (Figs. 7b and c, 8b and e). This distinctive difference between the radial and non-radial forces indicates that they are out of axial angular phase in a limiting effect on the vector sum totals of these forces as an expression of the vortical power (Fig. 2). Such an effect in variations of directions of total mutual force between gyres with relative axial angles would be closely relevant to the location and relative movement of tornado vortices adjacent to storm cell vortices in geographic plan, as well as to effects in the generation of tornadoes cited next.

As with radial forces, abrupt changes of level and reversals of direction of mutual forces, toward or away from coaxial alignment (Fig. 9a and b), are prominent in these data, as well as in the data of asymmetric vortices (Fig. 10a, c, and d). These limited data have implications for further study in the possible augmentation of tornado or waterspout vortices by the turning of short (somewhat conic) sections of horizontal cylindric gyres along local frontal boundaries as proposed in Project Vortex (Rasmussen *et al*., 2000; and Ziegler *et al*., 2001). These previously undescribed vortex re-orientations and abrupt reversals can also apply to flight conditions between vortex trails at busy airports and near developing micro-bursts in aircraft landing patterns, as well as to initiation and tracks of tornadoes and waterspouts in supercells or tropical cyclone rainbands on land or sea, to down-bursts in hurricane eye-walls, or to secondary and tertiary turbulences within and near storm cells generally.

9. Asymmetric force measurements

A limited number of force experiments with asymmetric vortices and with similar gyres in asymmetric positions were run in selected surface and immersed geometries that represent a small part of the scope of the possible geometries of research and operational interest. These include a coaxial base-to-point reference case, rotations of one axis from that reference and from symmetric cases (Fig. 10), a 20 degree cone paired with a 30 degree cone (not shown), an active cone with an inert cone (not shown), etc. (Pairings of identical cones at different rotational rates were omitted as less probable in nature.) The data were too widely varied and too narrowly limited in each type for additional equation adaptation. [For more discussion, an expanded back up is to be posted on the prior reference website (Howard 2006) after this publication.]

However, this short series of asymmetric experiments also included the largest measurement of attractive mutual radial force of 324 grams (Fig.10c), at very near the limit of 340 grams for 1X peripheral velocities estimated in equation (3) of section 7. That occurred with an asymmetric 30 degree angular displacement at the CV of one 30 degree cone from the coaxial BI $-180°\alpha$ symmetric configuration. This force measurement illustrates most conclusively the distinctive downward pumping action of the spiral wave in the upper hemisphere of vortices, as discussed in section 6b on downward inflow over a weather supercell (per Figs. 8g and i).

Above the more powerful vortex of a hurricane the low downward flow velocities over a much larger area of the mesosphere would focus an exceptionally large total downward momentum into the center of the spiral wave disk in the normally more compressed stratosphere. In the lab surface gyres (Fig. 1b), at the point in the spiral disk of change of water flow direction to outward there is typically an upper axial opening of air above the disk of water down to the center of the cone base. Thus, in spheric gyres, especially in the flattened sphere of a hurricane, there would be little central back pressure to stop the focused downward momentum (and likewise, no solid drive cone barrier.) Down flowing air momentum could penetrate similarly to the lowest level of the spiral wave disk and through it to contribute to the peculiar conditions of clear skies and calm weather in the axial eye of a hurricane, in its lower hemispheric driving body. (Radar chaff displays could test this.) Such penetration would be aided by centrifugal evacuation of the humid air of the lower eye into the inner margin of the convective eyewall below the spiral disk, provided that the hurricane is in a static balance of its various flows.

Such penetration conditions would occur when the upper axis is coaxial with the lower level eye of the cyclone, as with the increase of eyewall intensity and wind velocity in Hurricane Camille during loss of forward movement. Lateral deviations of such a downflow out of the coaxial alignment would create a breakdown or opening of the hurricane eyewall. This could occur when less humid continental air is entrained near the eyewall on coming ashore and weakens the cyclonic rotation on one side. It may also arise from upper level winds which displace the upper hemisphere even with initial high rotational power. When the lower level cyclone is building vigorously and moving geographicly at a high rate in tropospheric air mass flow, the inertia of the upper hemispheric mass may result in a trailing tilt in the upper axis with enough narrowing and offset of downflow into the eyewall to disrupt and open it initially, but a reconnection of the axes could cause a second inner eyewall to form. These empirical predictive implications indicate further extensive research in mesospheric flows above hurricanes.

10. Conclusions

An experimental laboratory survey and measurement of the little known mutual forces between pairs of highly turbulent conic vortices, with definite secondary flow structures, finds new explanatory correlations with the mechanical features of several forms of violent weather involving single and multiple vortical flow regimes. Outstanding in this are: (a) A direct numerical correlation of the newly described point thrust (**PT**) force of single hemispheric or sphericly complete conic gyres with the descent of tornadoes from mesocyclonic storm cells against the storm updraft. (b) A directly predictive correlation of the newly described spheric flow structure of single conic vortices with the presence of mesospheric sprites and other extremely high altitude electrical discharges observed over storm supercells at night. (c) Further predictive correlation of those conic spherical gyre structures with the source in such supercells of large scale mesospheric gravity waves faintly illuminated in night airglow.

Other weather implications of the forces and complete structures of these turbulent conic vortices related to forecasts of surface danger on land and at sea, to planning for storm emergency responses and recovery efforts, and to safety of flight from high altitude to landing and take-off or other low level operations, include: (a) The findings of this note that there are numbers of different unstable conditions in which the measured mutual forces between vortices may change their force levels, reverse the force directions abruptly, or quickly re-orient the axes of gyres; which effects could be significant to rapidly changing vortex trails and microbursts near airports, to thresholds of tornado formation and to present uncertainties in the location of tornado vortices adjacent to large convective storm cells. (b) A predictive implication that the vortical structures and forces which explain the mesospheric effects observed over supercells should also occur over tropical cyclones and cause the otherwise anomalous doubled eyewalls and irregular openings of eyewalls often associated with catastrophicly damaging downbursts at the surface, as noted with references in the first paper of this series (Howard 2006). These correlations and implications indicate a potentially valuable opportunity and approach for several intensified field and laboratory research efforts toward more precise and thorough understanding of the most violent weather vortex forces at all levels of the atmosphere.

These weather correlations and implications are made possible by laboratory measurements of vortex forces with a wide selection of combinations of the numbers of variables that control forces of single and dual turbulent conic gyres. First generation empirical equations are derived which numericly define these multivariate forces and their scaling parameters, as well as describing a newly found and significant **PT** force exhibited by single conic vortices generally. Major elements of turbulent conic vortex flow structure, vortex power levels, and vortex energy storage are included. There are new implications to turbulent boundary layers.

ACKNOWLEDGEMENTS

Particular appreciation is due to Matthew Clark, who had his students in woodworking at the Okaloosa County Vocational and Technical School (in Fort Walton, Florida) turn the four large 30 degree cones out of maple wood at no cost for these exploratory experiments. I must also thank Richard G. Henning of the US Air Force Weather Detachment at Eglin Air Force Base, Florida, Professor Keith G. Blackwell of the University of South Alabama at Mobile, Alabama, and D. D. Sentman of the University of Alaska Geophysical Institute at Fairbanks, Alaska, for discussions of weather effects which provided helpful background for this paper. Dr. Sentman's courtesy in providing by e-mail the video of sprite and gravity wave activity over a Nebraska supercell cited in section 6 is especially deeply appreciated. Again I thank Fred E. Howard, III, for a critical review of the text and many constructive comments on the general physics involved, and H. Blevins Howard for special support on computer equipment and software used herein as well as for monitoring internet weather data sources. I continue to owe much special appreciation to Cheryl Mack, senior librarian, and Christi Rountree of the US Air Force Armament Laboratory Technical Library, and likewise now to their new librarians, Eleanor Baudouin and Michael Jackson, for excellent and patient assistance of long standing with searches of the background literature.

REFERENCES

Fujita, T. T. 1992 *The mystery of severe storms.* WRL Research Paper 239 (A bound monograph). Wind Res. Lab, Dept. of Geophys. Sci., Univ. of Chicago, IL, USA. 298 pp.

Fujita, T. T. and Arnold, B. 1963 *Preliminary result of analysis of the cumulonimbus cloud of April 21, 1961.* MRP Res. Paper 16, Univ. Chicago, IL, USA. 16 pp.

Howard, F. E., Jr. 2006 Turbulent conic lab vortices scale to wide ranges of violent natural actions: I. Lab gyre flow structures define tornado inception sites in Hurricane Ivan. (Being condensed for publication.) (Website: www.electron-particlephysics.org)

Lyons, W. A., and Armstrong, R. A 2004 A review of electrical and turbulence effects of convective storms on the overlying stratosphere and mesosphere. Am. Met. Soc. Symposium on Space Weather, AMS Annual Meeting, Seattle, WA. Published on conference CD, 6pp

Lyons, W. A., Nelson, T. E, and Fossum, J. 2000 Results from the SPRITES '99 and STEPS 2000 field programs. Preprints, 20th Conf. on Severe Local Storms, Am. Met. Soc., Orlando, 4 pp

Markowski, P. M., Straka, J. M., and Rasmussen, E. N 2002 Direct surface thermodynamic observations within the rear-flank downdrafts of nontornadic and tornadic supercells. *Mon. Weather Rev.*, **130**, 1692-1721

McWilliams, J. C 1984 The emergence of isolated, coherent vortices in turbulent flow. *J. Fluid Mech.*, **146**, 21-43

Melander, M. V., Zabusky, N. J., and McWilliams, J. C. 1988 Symmetric vortex merger in two dimensions: causes and conditions. *J. Fluid Mech*. **195**, 303-340

Prandtl, L., and Tietjens, O.G. 1957 *Fundamentals of Hydro- and Aeromechanics.* Engineering Societies Monographs. Dover Publications, New York, USA

Rasmussen, E. N., Straka, J. M., Davies-Jones, R., Doswell, C. A. III, Carr, F. H., Eilts, M. D., and MacGorman, D. R. 1994 Verification of the origins of rotation in tornadoes experiment: VORTEX. *Bull. Amer. Met. Soc.*, **75** (6), 995-1005

Rasmussen, E. N., Straka, J. M., Richardson, S., Markowski, P. M., and Blanchard, D. O 2000 The association of significant tornadoes with a baroclinic boundary on 2 June 1995. *Mon. Weather Rev.*, **128**, 174-191

Rasmussen, E. N., and Collaborators 2001 29 June 2000 STEPS tornado near Goodland, KS. Severe Storms Research Papers, Cooperative Institute for Mesoscale Meteorological Studies, Oklahoma Univ., Norman, OK, USA

Riccardi, G., and Piva, R. 1998 Motion of an elliptical vortex under rotating strain: conditions for asymmetric merging. *Fluid Dynam. Res*., **23**, 63-88

Sentman, D. D., Wescott, E. M., Picard, R.H., Winick, J. R. Stenbaek-Nielsen, H. C. Dewan, E. M., Moudry, D. R., Sao Sabbas, F. S., and Heavner, M. J. 2003 Simultaneous observation of mesospheric gravity waves and sprites generated by a midwestern thunderstorm. *J. Atmos. Solar/Terr* **65**(5), 537-550 (Also circulated as a 2002 preprint.)

Smyth, W. D., and Peltier, W. R. 1994 Three-dimensionalization of barotropic vortices on the *f*-plane. *J. Fluid Mech*., **265**, 25-64

van Delden, A. 2003 Adjustment to heating, potential vorticity and cyclogenesis.

Q. J. R. Meteorol. Soc., **129**, 3305-3322

von Seggern, D. 1993 CRC Standard curves and surfaces. CRC Press, London

Ziegler, C. L., Rasmussen, E. N., Shepherd, T. R., Watson, A. I., and Straka, J. M 2001 The evolution of low-level rotation in the 29 May 1994 Newcastle-Graham, Texas, storm complex during VORTEX. *Mon. Weather Rev.* , **129** (6), 1339-1368

FIGURE CAPTIONS

Figure 1a. The measured data and derived equation (1) for the highly turbulent gyre diameter (*GD* in Fig. 1b), in centimeters for conic water vortices (centrally driven by 30° and smaller 20° cones), versus the peripheral velocity in cm s^{-1} of the base of each cone at various drive gear ratios from an electric motor (which was overloaded by the larger cones from the 2X to the 4X gear ratio. Gear ratios other than 1X direct also have losses in an added two-gear train.) The *GD* is the scaling parameter for subsequent force effects. The three point crossed line in the 30° cone's plot with 2X drive gear ratio is for a 4:1 variation of water viscosity with heat and sucrose sugar solution.

Figure 1b. Schematic planform and partial vertical section of a highly turbulent surface hemivortex driven by a 30° cone at 1X direct drive with a cone peripheral velocity of 872.9 cm s^{-1}. The cone is in left hand rotation at its base, which is at a standardized one diameter depth. The subplanar (slightly conic) disk of the centrifugal spiral wave planform, and each spiral wave in the disk, grow in turbulent thickness out to the *GD* and subside beyond it into smooth outward waves, with water particle currents indicated by arrows. For simplicity, only the cores of spiral waves are sketched, but the complete waves fill the intervening spaces. In the side elevation view, water from the surrounding surface tumbles in to the center over the top of the spiral wave disk and down to the top of the cone to feed the centrifugal outflow from above. Below the 14 degree elevated bottom of the spiral disk, water spirals more smoothly up and out from the sides of the cone, while also feeding the top of a toroidal gyre around the cone's body.

Figure 2. Electric drive power and stored energy of the same water vortices as in Figs. 1a and b are directly proportional to the dotted curve for ratios to a fixed reference of similar volumes of water within the *GD* diameter.

Figure 3. Measured data and derived equation (2) for axially directed point thrust (**PT**) of single conic vortices as a function of peripheral velocity of the two types of drive cones in water. Subtraction of components of this force from total measured force between a pair of conic vortices is an essential correction for mutual radial force between the gyres. This newly demonstrated **PT** force, inherent in all conic vortices, now accounts for the descent to the earth against storm updrafts of the conic body of a tornado. (Note the same velocity scales as with *GD*.)

Figure 4a. Definition of the relative axial angles of vortical drive cones which are varied symmetricly. Axes rotated with points of cones inwardly opposed (PI) are at positive relative axial angles. Axes with bases inwardly opposed (BI) are at negative angles.

Figure 4b. Schematic view in the reference plane of the axes of two immersed, sphericly complete, symmetricly arranged conic vortices (reduced from the first note of the series, Howard 2006). The gyres are scaled for 30 degree cones driven at 1X direct ratio with +30 degrees relative axial angles and cone volume centroid separations of 2 *GD.* Only cross-section outlines of the major secondary flows are shown, i. e., the spiral wave disk (including the *GD*) and the two unlike toroids in the upper and lower hemispheres. The irregular side profiles account for abrupt changes of mutual forces with relative axial angle or *GD* scaled separation, which can be significant factors with helicopter and low level aircraft wing vortices, microbursts, tornado intensification, and other weather situations.

Figure 4c. Introductory samples of the variation (with radial separation of cone volume centroids) of radial mutual force $\mathbf{F_M}$ between two turbulent surface vortices, with vertical axes at zero degrees relative axial angle (and 13 degrees), electrically driven in either co-rotation sense or contra-rotation at 1X direct drive gear ratio by two 30 degree cones in the curves to the left, or by two smaller 20 degree cones in curves to the right. There is a common scale in *GD* units of separation for the two cone sizes, plus separate cm scales and grams force scales. Note the reversed double arrows for abruptly unstable force changes in both direction and amplitude. See text for details.

Figure 4d. These two curves from one equation (15a) show the general mean trend of empirical data herein on the isolated radial mutual force between symmetric gyres versus radial separation on the turbulent *GD* scale over the full range of variable conditions in the prior figures and sections 2, 3, 5, and 7. Note that the radial mutual force between co-rotating symmetric conic gyres typically reverses direction from repulsion to attraction as separation is reduced in the vicinity of 3/4 of the common *GD*, with variations. The basic coefficient equation (15a) is always adapted for use in the force equation (4) by modification for specific conditions. Equations (5-14) multiply these two curves vertically. Equations (15b-24 and 10b) modify the curves horizontally. See text.

Figure 5a and b. These simple equations, for adaptation of force coefficients to contra-rotating pairs of symmetric gyres, are controlled by a lengthy series, contained in the text on equation (5), which gives the curves their peculiar shape to adapt the equations to sharp changes in forces when major features of the gyres (Fig. 4b) are exactly aligned. (The ripples in the horizontal lines may be disregarded as unimportant consequences of curtailing the length of the series to the minimum required in view of the experimental uncertainties.)

Figure 5c. The simple general equation (7) function of relative axial angles between certain co-rotating pairs of gyres. It is also widely modified in a variety of changes for other adaptations of the radial mutual force equation under the substitution rules of Table 1.

Figure 5d. Two variations of a major impulsive exponential equation (11) adapting to very large peak repulsive forces between co-rotating symmetric surface vortices in the lower range of relative axial angles. Note that the equation (13) for negative relative axial angles with bases inward (BI) is also used for a few cases of positive angles with points inward (PI) in the range of about 100 to 150 degrees rather than the low PI curve in that range.

Figure 5e. The graph of equation (12a) for an adaptive factor for whole cone body angles used in a few conditions in the calculation of radial mutual force between symmetric surface vortices. This factoral coefficient is graphed versus the absolute relative axial angles between cones, which is the other principal variable in the equation (12a) as a factor in equation (12).

Figure 5f. The very few definite radial mutual force data versus cone separations from limited comparisons of levels of surface roughness in viscously coupled driving of conic vortices with small 20 degree cones.

Note: Figures 6 & 7 are intended to be shown on facing pages for direct and ready visual comparison of the difference between the groups of graphs displayed. The scales of display should be the same, for full page on Fig.7, with the extra space at the bottom of Fig. 6 for the short captions on both 6 and 7. If the captions are too long to fit, they should be reduced in length to fit this space, as in alternate captions shown further below.

Figure 6. (above) Radial mutual force data in grams, between symmetric pairs of surface hemivortices in lab water, and adapted equation curves per Table 1, versus cm separation of centroids of drive cones with points inward at positive relative axial angles of 30° cones unless 20° is noted, co-rotating in upper reversibly repelling curves (solid

symbol), counter rotating in lower attracting curves (open symbol). Compare conditions between plots and with Figs. 7 and 8. Note abrupt unstable reversals of forces. See text and Fig. 8 for details.

Figure 7. (opposite) Radial mutual force data and equation curves for immersed pairs of spheric vortices with wider variation than in Fig. 6. of drive gear ratios, peripheral velocities of cone bases, symmetric relative axial angles, and water viscosities. See text for details.

Alternate Shorter Caption for Figs. 6 & 7. (NOTE THAT THE LAST LINE AND A HALF MAY BE CUT IF SPACE REQUIRES, since this info is also in other Fig. captions referenced.)

Figures 6 (above) and 7 (opposite). Compare symmetric pairs of surface and immersed vortices respectively for variation in grams of radial mutual force data and equations with centimeters separation of 30 and 20 degree drive cones, under varied drive gear ratios, relative axial angles, and water viscosities. Note abrupt unstable reversals of forces. See text and captions of Figs. 1a, 4a, b, c, d, and 8 for details. Equations are per Table 1. Pairs for upper curves (solid symbol) are co-rotating; for lower curves, counter rotating. Positive forces repel; negative forces attract.

Figure 8. Empirical radial mutual force $\mathbf{F_M}$ data and equations (per Table 1) for symmetric pairs of fully immersed conic vortices (and a few surface cases) driven in water near the lab standard by 30° cones at negative relative axial angles (α), i. e., with cone bases inward BI and opposed (per Fig. 4a) rather than points inward PI (Figs. 6 and 7). Drive gear ratios (1X, etc.) and consequent cone peripheral velocities (V_P) are widely varied. Data are graphed in grams force versus centimeters (with the *GD* indicated) of radial separation of volume centroids (CV) of the driving cones. Negative forces attract or move the two cone CVs closer together. Positive forces repel or increase the initial separation of the two cones. Pairs of gyres for upper curves (solid symbol) are co-rotating or in the same hand of rotation on the bases of the right circular cones; pairs for lower curves (open symbol) are counter rotating. Double and triple down arrows at minimum separations of the internal CVs of the cones indicate that the cones (or parts of the pendulum mounting gear) have been attracted together (hit and stick together) so forcefully that the total force is greater than can be measured by the pendulum deflection. Reversed arrows between two points indicate abruptly unstable changes of direction and amount of force. Such abrupt reversals of force direction correlate directly with

hazardous uncertainties of weather/forecasting that bear on safety of flight, surface damage, etc. See text. See Fig. 9a for same 30 degree data scales in larger size.

Figure 9a. Crawl-corrected data on fully immersed mutual forces in grams $\mathbf{F_P}$ perpendicular to radial mutual forces $\mathbf{F_M}$ and in the reference plane of symmetric cone axes, versus centimeters of CV separation of the pair of drive cones. Compare with Figs. 6, 7, and 8. The counter rotating pairs lock in the initial coaxial alignment, and their considerable attractive (negative) force toward coax alignment could not be readily measured. As in all other figures, reversed arrows indicate an unstably abrupt reversal or change forces. Also note especially the contrast in peak values of force with the 150° to 180° data of radial force trends in Figs. 6, 7, and 8 at the same drive gear ratios and consequent V_P, and see the text discussion. Again, these effects indicate uncertainties about vortices involved in tornadoes and other weather phenomena.

Figure 9b. Some samples of sums in grams force versus cm separation of gyres for non-mutual crawl and mutual lateral force, both perpendicular to the vertical reference plane of the two cone axes, in water surface vortices when guides to eliminate these lateral pendulum deflections were removed briefly. Some instabilities are noted by the reversed arrows. Counter rotating gyres tend to maintain coaxial alignment of axes in plan projection with some unmeasured negative force greater than zero, with two distinct exceptions.

Figure 9c. Lateral crawl forces in grams in the horizontal plane of the immersed pendulum rig versus the peripheral velocity in cm/sec (and drive gear ratios) of a single 30 degree whole angle cone with two gears in the drive train. These data are used for correction of mutual forces between pairs of cones where noted. See text for discussion.

Figure 10. A small sampling of the much greater scope of forces between asymmetric pairs of gyres which might be investigated by any additional experiments. The asymmetric cases often are not consistent with the trend of symmetric experiments in the prior graphs. The angles listed in the subfigure titles are pivoted around the dots which indicate the centroids of the volumes of the cones which drive these conic vortices. The very high attractive force in counter rotation at small separation of the top configuration in Fig. 10c (consistently here with Figs. 8g and i) correlates with variations of eyewall structure in tropical cyclones and upper level disturbances over storm supercells. (See sections 6b and 9.)