(TITLE & ABSTRACT AFTER REVIEWER COMMENTS)
(BACK-UP DETAILED DISCUSSIONS)

SUB-STRUCTURE LAWS OF PARTICLE MASSES AND CHARGES ---A NEW SYSTEMATIC CLASSIFICATION OF SUBATOMIC PARTICLES

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ABSTRACT: *This research paper completes a systematic classification of subatomic particles (and anti-particles) by the way in which their masses, charges, and stabilities occur. The conventionally elementary particles are numericly divided in several new classes: By the number of different quantized masses with a single charge status in which each type of particle is found herein to occur; by two new cycling zonal groups of odd or even multiples of 1/3 fractional charge level (and by normal matter, with odd multiple zones negative and even multiples positive, versus anti-matter, with the new cycle zones reversed); as well as by the usual three families of zero, unit, and fractional charge; and, across these families and groups, by gradually phased, but distinctive new zones of relative cyclic regularity and stability, with the resulting general prevalences observed in nature. This coordinated classification system arises as a consequence of new analysis herein of how the two previously reported microquanta of positive and negative charge, each with the same uniform microquantum of mass, can combine under an empiricly derived power law of quantized mass and charge to compose the conventionally elementary particle/anti-particle families of the unit-charged leptons, neutral neutrinos, and fractionally charged quarks (LQ). Interaction of that power law with a related exponential law of mass and charge coordinate cyclic distributions, quantally derived herein, constrains each specific type of LQ particle from the quantized optional mass values which are* <u>not</u> *available to that kind of particle within the quantal*

cycles of this system of laws, and positively constrains each type of particle to the regularly cyclic, specific mass/charge values which it can possess in the system. This systematic classification of particles independently confirms Particle Data Group (PDG) empirical uncertainties on the Standard Model (SM) single masses of both the quark and neutrino families of massive particles. That confirmation arises from new findings herein that, rather than the single SM mass in each case, there must necessarily be both multiple cyclic neutrino masses and two well constrained cyclic quark masses within the wide PDG empirical uncertainties, as respectively appropriate for each of the SM cases of the conventional three neutrinos and three light quarks. These general findings are substantiated by specific demonstrations in the prior paper of direct processes for neutrino mass "oscillation", and for construction of two major PDG observed classes of proliferating baryon hadrons by their composition from the quasi-elementary particles re-defined herein. These findings also predict substantiation of two or more cyclicly constrained masses for each of the three SM heavy quarks. There is an implication that the lowest mass neutrinos may be the most stable and numerous. The existing substantiation extends the effects of the systematic mass/charge zonal classification of the quarks/anti-quarks, as a family of the LQ particles, to the massive empirical hadron particles, which are composed of the quarks. Thus, a new, simple basis is given for a more complete eventual substantiation of how the masses and charges of all the massive sub-atomic particles of observable matter in their exotic proliferations, from the cosmic microneutrinos to the multiquark hadrons, occur as a single, continuously linked and systematicly regular structure of quantized composite mass with conserved charge. What the more fundamental nature of the two enumerated microquanta building blocks can be and why they work together in a way that enables this structuring of the known massive *particles and their varying mass and charge combinations also remains to be addressed. Zonal relations within this classification define a more significant grouping of the LQ particles than by the conventional three "generations."*

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(ORIGINAL SUBMISSION TITLE, ABSTRACT, AND LONGER DETAILED DISCUSSION)

SUB-STRUCTURE POWER LAWS OF PARTICLE MASSES --SYSTEMATIC CONSTRAINTS--A NEW CO-ACTIVE LAW --LEPTON-QUARK-HADRON STRUCTURAL CONTINUITY

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ABSTRACT: *A previously reported new power law for mass/charge sub-structures in conventional elementary particles described regularly quantized numerical options in mass values matching the empirical data of the authoritative Particle Data Group (PDG), and the Standard Model (SM), for all the lepton and quark/anti-quark (LQ) elementary particles. The basis for the law was a generalized universal mass micro-quantum with 1/6 positive or negative charge. Specific mass options were correlated with known particles primarily by consistency with the empirical data. Those correlations were substantiated and extended across the sub-atomic spectrum of massive particles by examples of SM/PDG hadron particle groups structured from the named components in accordance with the new law.----This research note newly derives positive restriction of each specific type of LQ particle to its specific unique mass value by constraining the usual (U) range of quantized optional mass values which can* <u>*not*</u> *be occupied within this power law system by that kind of particle. A new exponential law is derived which re-interprets and acts* *jointly in this constraint with the original mass power law. The new constraints complete a definition of the systematicly regular numerical mass relations that are quantally available between all LQ particles and, consequently, between all massive types of particles. Thus, a new, simple basis is given for approaching the masses and charges of all massive sub-atomic particles as a single, continuously linked and systematicly regular structure of quantized masses with conserved charge. Zonal relations imply a more significant grouping of LQ particles than by the conventional three "generations."*

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Key Words: Leptons, Quarks, Neutrinos, Composite Particle Sub-structures, Mass Power Law, Regularity of Mass Progressions, Massive Sub-atomic Particles, Hadrons

INTRODUCTION---This research note began as a possible different way of graphing the data from a prior precedent note (Howard, 2005). That paper described a power law for the numerical regularity of quantized LQ particle masses and conserved charges, in consistency with the PDG empirical data.(Eidelman et al., 2004) The basis for the law was a generalized universal mass micro-quantum with 1/6 positive or negative charge, of which LQ particles would be composed in pairs of these components. Through that law the masses of LQ particles (such as the electron or quarks) can be computed as composite particles by applying a function of: (a) the sixth power of the number of their components, which are linked in pairs; and (b) the ratio of the number of pairs of components with a net charge (in each of those pairs) to the total number of pairs (including the neutral pairs.) The new graph presented herein could possibly reveal new relations between particle masses/charges beyond the scope of the original note. Potentially, broader insight might contribute further systematic distinctions between those regular estimated mass/charge values which are optional under the power law but cannot be occupied by the various SM types of LQ particles, and those systematicly specific quantal values which are already considered occupied by correlation in the prior note with empirical data for these particular sub-atomic particles from the internationally verified PDG biennial report (Eidelman et al., 2004).

That possibility is confirmed here in Figure 1 as analyzed in subsequent figures and discussion. A new exponential type of law, derived from the graph, re-interprets and acts jointly with the original power law to confirm the computed LQ mass/charge relations. The dual, co-active law result in confirmed LQ masses is then more positively extendable to massive sub-atomic particles in general.

The underlying regularity of the quantized LQ particle masses and electric charges under the original equation (6) of the prior reference (Howard, 2005) is applied throughout its usual (U) range covering the SM data listed by the PDG (Eidelman et al., 2004). The new law presented herein particularly confirms the numerical regularity of quantal masses and charges of these conventionally elementary particles. All the masses for the LQ particles were estimated under the power law in the prior note and were found essentially consistent with the PDG listed empirical masses and their range limits or uncertainty limits (Eidelman et al., 2004). (Among more than 20 optional mass estimates for the 12 obligatory types of LQ particles, inclusively subsuming anti-particles of nominally equal masses, there was only one deviation from the PDG accredited uncertainties of empirical values of as much as 3.3%, three of less than 2%, and two of less than 0.5%. Over two thirds of the quantally exact estimates did not deviate from the PDG empirical values.) The usual U range of application of the power law considered here omits only the PDG mass limit of the electron neutrino at its extremely (E) small mass value isolated at five orders of magnitude below the smallest definite PDG mass of any other LQ particle, and two orders of magnitude below the U range of this law.

As in the prior note, there are additional implications beyond the initial objective that may contribute further. In the prior note those implications included substantiation of the two estimated values found for each of three quark sets. Substantiation of these six estimated LQ particle mass values was accomplished in introductory trial examples by using these specific quark set mass estimates to account for the peculiar distribution and numbers of PDG empirical mass values in two proliferative groups of hadrons known to be constructed from these quarks. In this current note, new implications include a possible re-grouping of the LQ particles that is different from the conventional three "generations." The consequences of the numerical laws observed here do not always follow that grouping. Otherwise, consistency with the SM is largely maintained. The joint co-active law derived here provides a more firmly constrained and well defined mass system basis for a much stronger additional implication than before with only the original power law. On this simple, easily comprehendible, and introductorily substantiated basis between the two laws, both essentially derived originally from the empirical data on the quark compositions of hadrons, the empirical mass distribution of all the massive sub-atomic particles can now be newly approached for more extensive analysis as a single, readily understood, continuously interlinked, and systematicly regular structure of uniquely quantized masses with conserved charge.

Describing a new graph of prior LQ particle data---Figure 1 displays the prior estimated masses (Howard, 2005) of SM LQ particles over their ten orders of magnitude of the usual (U) range of these masses as defined in the reference. In this new graphical form, the prior note's estimates of LQ particle masses m_p in electron-Volts are shown in log format as the base against which to plot the log of the charge factor *F* in the power law for each of these masses as stated for the U range in Equation (6) of the reference (Howard, 2005):

$$m_p = (2m_u/3)\, N^6\, F \tag{1}$$

where

$$F = \{0.5 + (n_{\pm}/n)\} = \{0.5 + (p/n)\}, \tag{2}$$

N (which equals 2*n*) is the number of universal charged component quanta with charges of + 1/6 or – 1/6, m_u is the mass of each component quantum at 10.9525 electron-Volts (as calibrated in the equation against the PDG mass of the electron rounded to 0.511 MeV), *n* is the number of the usual organized pairs of all such components including the neutral pairs (+ 1/6 and –1/6), and $n_{\pm}$ (or for greater distinctiveness hereafter in this note, $p \leq n$) is the number of charged pairs of components (whether charged + or – 1/3 per pair of like 1/6 charges.) The value of *F* ranges from 0.5 where $p = 0$, to 1.5 where $p = n$. The plot includes the quantized options under the power law of all potentially reachable U range masses for LQ particles, though not all of these optional mass sites can be occupied by the considerably smaller number of LQ empiricly observed SM particles listed by the PDG.

The empty mass sites that are not occupied (Fig.1) are noted on the lines of plots by a dot without an additional symbol. There is a straight plot line on the log-vs-log plot for each number of quantal components (and pairs of them) denoted, for example, as 6(3) or 24(12) for the line. All of these lines have the same parallel slope on the plot, due to the same ratio of 3/1 or factor of 3 between the high mass and low mass ends of each line in accordance with the range of the equation (2). The steepness of the lines' slope is largely due to the difference in scales for the two axes. The entire *F* scale covers only its factor of 3, slightly less than half an order of magnitude. If this scale were not expanded (for open display of the particle sites) by a ratio of about 9 to 1 (+5%) over the mass scale, the lines would slope at 45 degrees for the 1/1 exponential slope of the first power of *F* in the equation (1). The sixth power of *N* components takes effect in this graph format by spreading apart the numbered lines for pairs of components on the mass axis (rather than by the straight line exponential slope of 6 which would appear if mass were plotted in log scales against *N* without the cyclic effects of *F* or at a fixed value for it such as the central value of 1. That value would be very close to the average, within 0.01%, for all the estimated sites.) (Howard, 2005.)

Each line has a mass point at its low mass end denoted as 0/3 or 0/12, etc., for zero charged pairs, meaning all pairs neutral, having a plus 1/6 and a minus 1/6 charge, with each of these charges on a separate mass component in each pair. In ascending mass order on each pair number line in accordance with the equation (1) as controlled by the equation (2) for factor *F*, there is then a logarithmicly spaced mass point site for each step (to be counted) in progressively charging the pairs on that line to fully charged with no neutral pairs remaining. Starting with the zero charged points, there is always one more point on each line than the number of pairs the line is named for. Thus for the 3 pair line marked 6(3), there are four points on the line, a mass value location for 0 charged pairs denoted 0/3 with a $p/n = 0$, a mass value for 1 charged pair denoted 1/3, a site for 2 charged pairs denoted 2/3, and a fourth point for 3 charged pairs denoted 3/3 for all pairs charged and a $p/n = 1$.

Note that on this steeply slanted 6(3) pair number line the SM muon neutrino is shown (by a small filled circle symbol for a primary neutrino value) as occupying the 0 charged pairs 0/3 point (or site) at 0.1703 MeV estimated mass from the referenced paper (Howard, 2005.) As noted there, this mass value is well correlated with the PDG upper mass limit for that particle of <0.19 MeV (Eidelman et al., 2004.) The SM electron, with a net electric charge of 1, occupies the fully charged, 3 charged pairs 3/3 point (large open circle symbol) at the 0.511 MeV mass value. (In the prior paper this value, rounded from the very precise PDG value, was used, as noted earlier,to calibrate the mass quantum as a basis for all the calculated mass estimates.) The 2 charged pairs site 2/3 is indicated as an optional alternative (small open circle) tau neutrino estimated mass point at 0.3974 MeV (with a non-intuitive net charge level of 0, as explained in the next section.) The 1 charged pair site 1/3 at 0.2839 MeV mass with the only net charge level it can have, that of 1/3, is necessarily empty. Only a quark/anti-quark (cross symbol) of the LQ particles could have such a charge level, and there is no suitable (hadron component) combination for such a particle in this entire low mass zone below the 4 MeV lower mass limit for the down quarks or anti-quarks (in correlation with the SM/PDG current mass system for quarks/antiquarks.) A net charge level of 2/3 might be occupiable down to a (hadron component suitable) level of 1.5 MeV mass as with the up quarks and anti-quarks, but that net charge level is not obtainable in this power law system at a 1 charged pair site such as this one. (This is explained more fully in the next section.)

It thus begins to be evident before completing the initial inspection of the graph that any particle's specific occupation of optional mass/charge value sites in the figure is constrained by at least two (or more) different types of zones, both by influences within the power law equation (1) system, and by outside influences with which it must correlate. This factor of double (or multiple) zonal constraint implies that there may not be a single numerical relation or equation that fully defines all the occupied optional mass sites as distinct from the unoccupied or unoccupiable optional sites. It is more likely from the start that a group of relations including the power law will be necessary to make that distinction zone by zone, and some of the specific relations may not be immediately evident or fully definable within this scope. Thus, only a somewhat incomplete contribution to such distinctions, rather than a fully definitive distinction, may finally be available through review of the graphed data. There may be residual zones of optional mass sites for which the distinction cannot be considered fully resolved even though distinctly implied. The end result of specific mass/charge site locations in the graph for each and all empirical LQ particles may not have the absoluteness of a single new equation with a root for each mass assignment; rather, it may be definable by convergence of several sufficient necessities, of which one would be the power law equation (1).

To proceed with the initial description of the new graph (Fig. 1): Where there are an even number of pairs on a line (with the consequent odd number of mass value sites), as with the 8(4) line for 4 pairs, the balanced point with half of the pairs charged, as at 2/4 with 2 charged pairs out of 4, falls on the central graph coordinate line for $F = 1$, as with the sum of 0.5 and 2/4 in the equation (2).

Since the number of mass points becomes inconveniently large and densely crowded on the graph lines beyond the 16(8) pair number line with eight pairs, the empty mass point dots are omitted in the higher mass lines. However, each occupied mass point is denoted as by 4/12 for the 4th charged pair out of 12 pairs for the 24(12) line. The mass estimate may thus be calculated quickly in Equations (1) and (2) if desired, or read very approximately by log estimates on the particle mass axis (Fig.1.), or looked up by pair and charge designations in Table 1 of the reference (Howard, 2005). It is also notable that some of the higher mass lines for pair numbers are not completed across the diagram, nor even indicated on the graph margins at the highest mass levels, wherever there is no SM particle on that line. The line and all its optional value sites are then empty. (However, the steps in mass increase due to increase of charged pairs for the omitted sites and lines must be counted for the total number of steps and mass value sites up to the next higher LQ particle occupance of an optional mass/charge site as described in the next section.)

The apparent crowding of lines and mass sites in the higher mass areas is due entirely to the very condensed logarithmic scale of orders of magnitude there. In any linear plot the possible sites would be crowded at the low mass level and very widely spaced at the tenth order of magnitude in mass. At this high mass level then, the occupied sites are comparatively very thinly dispersed in a quite thinly dispersed set of a large number of possible sites. Up through a mass of 10^7 eV about three quarters of the sites are occupied, but above that range the fraction of sites that are occupied falls off rapidly until, at the highest mass levels, only about 2% of the available sites are occupied by optional mass estimates for known particles. There are a total of 464 numericly regular possible sites available from the lowest through the highest pair number line on the graph. Thus, the lower order-of-magnitude mass range contains the only pair number lines that have large fractions of occupancy by the less than three dozen estimated optional mass values that correlate with, or fall within, the upper and lower mass limits for the 12 PDG/SM listed and empirically known LQ particles.

At just above the 10 MeV (10^7) mass level, where the 12(6) pair number line begins at 0/6, more exactly from 10.901 MeV and higher, the pair number lines are no longer clearly separated from each other in mass level. The lines begin to overlap progressively more and more as mass increases, and the separation of lines is much less than the accumulation of mass increase with the slope. This means that a continuing step count from the high mass fully charged mass point on a line to the next higher pair number line may actually decrease in mass values with the increased step count to the low mass end of the next higher line. In the higher pair number lines the overlap is such that only the most fully charged mass points can have a net mass increase over the highest masses of the adjacent lower pair number line. This may have interesting consequences in superficial appearances of non-regularity of particle masses in particle groups to be discussed later.

Understanding the new graph---Steps in both charge and mass increase are necessarily taken together along the pair number lines of Figures 1and 2. (Functions of the number of these steps between particle mass sites will be involved in equations to be derived in a later section.) The steps of increase in mass and charge are counted at each mass value point (whether occupied or not) and along each line between the start and the finish of any count between two particles if it includes more than one line. (This is true even for the points and the lines not actually shown on the plot, but inferred from the line pair number or site designations that are shown.)

As an example (Fig. 2), start from the electron *e*. *(*Its site on the 6(3) line indicates that it is estimated to have 6 component quanta contained in 3 pairs. The site name 3/3 indicates that 3 out of 3 pairs are charged rather than being neutral. Since this permits a net charge level of 1, for the particle to be an electron with a empirical charge of −1, each of the 3 pairs must have a charge of −1/3, made up of two component quanta of charge −1/6 each, rather than being neutral with −1/6 and + 1/6 component charges neutralizing each other to 0 charge for any given pair of components.) From the electron site to the site of the smallest up quark u at 2/4 in the 8/4 line there are 3 steps. First, there is a flyback step to the low *F* border of the graph on the next higher line 8(4) at the 0/4 point with 0 charged pairs. (See the several dotted flyback steps indicating this process in Figure 2.) Then there is a step to the 1/4 charged pair point, and next a third step to the 2/4 charged pairs point, at which the smaller *u* mass is found with a net charge level of 2/3. (Counting the flyback step from the fully charged pairs mass point at the high *F*, high mass end of one line to the zero charged pairs mass point at the low *F*, low mass end of the next higher line is essential in the subsequent derivation of an equation to calculate mass as a function of the full number of steps for charge and mass step count intervals on all lines.)

The second and larger u (up quark) mass option with the same 2/3 net charge level as the smaller mass u at 2/4 (whether +2/3 for the quark, or −2/3 for the anti-quark) will be found at two (even number) steps higher, 4/4, on the same pair number line as the lower mass. This even number interval may seem counter-intuitive. However, it necessarily takes an even number of changes of charged pairs, at 1/3 charge for each pair, to have the same net charge level at two points on a single line. That necessity does not apply in flyback stepping between lines from even to odd pair number lines, but does apply in stepping from odd to even pair lines.

This seeming irregularity is in reality quite regular because of the variable charge/pair combinations. At the zero step on all pair lines $F = 0.5$ and no pairs are charged. At this step each pair has two component quanta of −1/6 and +1/6 charge, which neutralize each pair within the pair, and the net level of charge for a particle at any of these mass/charge combination sites can only be 0. At the first step up in mass, one pair is charged either plus (++) or minus (—) for a net particle charge level of 1/3, whether plus or minus. At the second step up, another pair is charged either plus or minus for a net charge level of either 0 or 2/3. At the third step up, with 3 charged pairs, the combinations may give a net charge of either 1/3 or 1 of either polarity. At the fourth step up, with 4 charged pairs, the combinations may give a net particle charge level of either 0 or 2/3 again (A net particle charge level of 1 1/3 is not permitted in the SM nor empiricly observed in the LQ particles.) (Eidelman et al., 2004.) If there is a fifth step up on a line, with five charged pairs in any particle at that site, the combinations are back to net charge levels of 1/3 or 1 again (1 2/3 is also not permitted in the SM nor observed in the LQ.) And so forth, in a cyclicly repeating process. Every pair number line proceeds in the same way, with one more site on each line than its pair number. This charge quantization cycle controls where the various particle charge characteristics can appear in the mass plot as specificly as the available quantized mass at each site. The control is based both on the total number of paired component quanta and on the ratio of the number of charged pairs to the total number of pairs. Both must determine any set of estimated charge and mass characteristics for any site shown on this graph to be occupied by an LQ particle (and all LQ particles are included here.) With these understandings, some systematic regularities may be explored.

INTERPRETING THE NEW GRAPH FOR PARTICLE MASS RESTRICTION ZONES---In Figures 1 and 2 the 0 step mass value points of all the pair number lines lie along the low mass horizontal border line for the value of $F = 0.5 + 0$, where any particles can only be neutral with 0 (zero) net charge, as with neutrinos. In Figure 2, in accordance with the systematically cyclic net charge determination process just described, a curved line of medium length dashes connects all of the first steps above the 0 charge level on each pair number line.

These are all sites with 1 charged pair, at which the net charge level of particles occupying the site can only be 1/3. Likewise, a curved dashed line connects all the 2 charged pair sites where the net charge levels of particles can only be 0 or 2/3. As noted earlier each of these quantal increases in the number of charged pairs also has a necessary quantal increase in mass. Particles on the dashed line for 3 charged pair steps, which passes through the electron site, (with the associated masses) can only have net charge levels of 1/3 or 1. These possible net charge levels for any particles occupying the sites alternate in zonal cycles in this way across the graph of particle masses in logarithmicly spaced diagonal curves in progression from curve to curve toward the corner of the figure (Fig. 2) where the highest mass pair number lines have the most charged pairs and highest mass step sites.

In the 6th order equation (1) of the power law which creates the basic pair number line and site structure of these graphs (Figs. 1, 2), the specific mass values for particular known particles are not easily predicted by the equation. From its form alone, without selective correlations of its cyclicly permitted mass value estimates with the uncertainty limits of empirical mass measurements, particle site occupations might randomly appear anywhere in the systematicly regular, discretely quantized, optional mass/charge value points specificly permitted by the equation and graphed in the figures. (Each of these points is a true mathematical point without dimensions or tolerance, though the value may be known or specified only to a few significant decimal figures with an uncertainty due largely to rounding of values. Likewise, even though connecting lines are shown to clarify relations between points, there are no continuous intermediate values between specified points, such as varied probabilities of particle detection; in this mass/charge power law system only the specific quantized point values of mass and charge exist.)

However, the sites estimated to be occupied by empirically known particles are not random, as noted earlier. For any particular net charge of a particle, whether of a known particle or not, the charge element of the equation (2) in the *F* factor excludes the mass value sites on every alternate curved net charge line. (And every SM/PDG (Eidelman et al., 2004) particle in this system (Howard, 2005) has a quantized and definite net charge status even where the exact PDG empirical mass value is widely uncertain.) The reverse is also true, in the sense that the level of net charge in a particle can in this system only appear on the correct curved charge line where the particular level of charge is quantally accounted for (in conjunction with the particular quantally discrete mass.) Inherently, without reference to the empirical particle masses, there is at least a small zonal restriction of a minimum of 50% of the mass value site availabilities in this power law system for any particular kind of particle due to particle charge requirements marked by the curved lines.

THE NEUTRINO LOW MASS ZONE--- Within the above constraint that only 0 net charge lines (Fig. 2) are open to occupation by the 0 charge neutrinos, and otherwise constrained only by upper mass limits (without lower limits) in the PDG empirical data on the three types of neutrinos (Eidelman et al., 2004), there is a definite implication that neutrinos may occupy, and could exist at, every potentially neutral site in this U range with mass lower than the PDG mass upper limit of <18.2 MeV for the SM tau neutrino (as indicated by the small circles in Fig. 2.) That implication is directly consistent with providing a structural basis in the prior power law paper (Howard, 2005) for the empirical observations (cited there) of neutrino "mass oscillations." The implication is further substantiated by the numerous neutrino mass oscillation correlations with power law structure estimates in Example 4 of Appendix C of the reference.

This larger zonal implication on neutrino occupations of sites applies only for these lower mass neutral sites and differentiates them from those unoccupied optional neutral sites above the mass boundary limit. The closely matching estimate (Howard, 2005) of 18.169 MeV for the largest tau neutrino mass option appears as a small filled circle just below the zone boundary limit at 2/6 on the 12(6) line (Fig. 1, 2.) (Note that this highest mass neutrino site occurs with one smaller neutrino alternate, indicated by a small open circle, on the otherwise unoccupied 12(6) pair number line which begins at the 10.901 MeV boundary point of line overlap noted earlier.) By this previously substantiated implication, neutrinos would occur at other masses below the 18.2 MeV zone limit, not only on the 0 charged pair axis line, but also below the zone limit on the curved lines in dashes in Fig. 2 for the 2 and 4 evenly charged pair cases. Also, a more definite emphasis should be given to the particular filled circle site at 0/3 with a mass estimate of 0.1703 MeV just below the PDG upper limit for the muon neutrino of <0.19 MeV, as discussed in the basic reference note.

These alternate option sites occupied by neutrinos can have a balance of oppositely charged pairs for a 0 net charge level or, where there are sufficient pairs, an optional imbalance for 2/3 net charge. This occurs on the 8(4) pair line, where two alternate neutrino sites at 0 net charge marked by small circles are co-located in mass with the two up quark set sites at the 2/3 alternate charge level marked over the circles by large X symbols. (Additional relations between particle types to be discussed later also involve neutrinos.)

Overview of Masses of the Quark sets--- Overview of Masses and Charges of the Quark sets---The joint occupancies at the 2/4 and 4/4 mass sites of neutrinos at 0 net charge with the two members of the up quark set at 2/3 net charge is a normal result of the power law system in the net charge level curves. The up quarks/anti-quarks with their 2/3 net charge levels are already constrained to these two specific sites at 1.914 and 2.871 MeV by the PDG mass uncertainties of about ±45% around the mean of the upper and lower empirical mass limits of 1.5 to 4.0 MeV (Eidelman et al., 2004), and by the site mass substantiations in providing (with the down quark sites) a basis for the observed proliferation of the N nucleon hadron masses (Example 3 of Appendix C, Howard, 2005), as well as by both general constraints already considered. The reason for the empirical inability to reduce the large quark uncertainties is thus proven to be that multiple quark sites of a type are used by nature as hadron components.

Therefore, if these two up quark sites do not suffice under the power law system (Appendix C, Howard, 2005) to make all the observed hadrons which must have up quark components, then more optional sites on the same net charge level curves (such as sites 2/3, 2/5, or 4/5) but outside the PDG limits must be designated at some percentage level (of reduced probability) to complete this part of the natural function of composing hadrons. To what further extent these two mass sites near the present peak of probable presence in up quark composites are constrained by power law systematics from such dispersion is still to be determined herein. This is also the case in general for the other quarks.

Overview of Masses of the Quark sets---Aside from those directly implied occupations of neutrino sites, with only two of them closely connected to PDG listings of upper mass limits, there are also the implications of the prior reference (Howard, 2005) on quark set estimated masses. There it was noted that the width of PDG uncertainties on quark masses (Eidelmann et al., 2004) can be accounted for by the implication that at least two masses may be present for each PDG listed mass range for the quarks and their matched anti-quarks (in sets nominally of the same mass value.) Here that implication is extended (subject to further discussion) to all optional quark set sites (cross symbols) on the proper net charge curved lines of Figure 2 within or very close to the PDG listed uncertainty range of empirical mass for each quark/anti-quark set. This still includes only two sites each for the up, down and strange quark sets. But as seen in the figures, it increases to three sites for the charm quark set and four sites each for the bottom and top quark sets (several of which sites were not previously noted in the prior paper.)

As will be shown, in this view, the up and down quark sets have the simplest individual cases and joint mass/charge site relations of all the quark sets. Their mass ranges and sites lie along adjacent parallel pair number lines in the heavily emphasized lines of Figure 3. As indicated by the arrows and the vertical dashed reference line, these pair number lines slope positively upward toward higher charged pair ratios with increases of mass. The connecting lines between their two higher masses and their two lower masses complete a quadrilateral figure that is almost a regular parallelogram. The slightly different slopes of these two lines down to the lower F values of the down quark set are very negative. The highest estimated mass of the down set is well below the 10 MeV region at which the pair number lines begin to overlap. These features take significance from the sharp difference when the strange quark set lines are added in Figure 4, and even more significance later when the other set lines are added and regularities of mass site constraints are explored with Figures 5 and 6.

The emphasized line connecting the two estimated mass/charge sites of the next higher mass strange quark set in Figure 4 is nearly an order of magnitude above the line overlap boundary at 10.9 MeV. Within the PDG (Eidelman et al., 2004) upper and lower mass limits for the strange set, there are two correlated mass sites permitted by the power law on two adjacent pair number lines rather than on a single pair number line (as occurred for each of the up and down quark sets.) Consequently, the emphasized line for the strange quark set has a very steep negative slope. This is indicated by the down arrow and the vertical dashed reference line to correct for confusion of the eye in the presence of the sloped pair number lines. The negative slope of this mass site connecting line is generally typical of the three other higher mass quark sets, though not always in every section of their connecting lines (i. e., with the exceptions of the lower sections of the charm and bottom quark set lines in Fig. 5, which is to be considered later.) As will be seen, this change (across the region of the overlap boundary) of the slopes of the emphasized lines connecting the mass sites within each quark set, is significant to the question of whether the quark set masses are constrained into a distribution over the quite regular optional mass sites of the power law, with an additional, final and complete, distribution regularity of full constraint within the power law system to the specific regular sites they have previously been estimated to occupy.

Neither of the two pair number lines connected by the mass connection line for the strange set (Fig. 4) is adjacent to the single pair line for the down quark set. The upper and lower lines connecting the strange emphasized line with the down set, slope positively with increasing mass. The resulting second quadrilateral figure (quad) is very broad compared to the first quad and oppositely shaped. It is also distorted or warped away from any resemblance to a parallelogram of regularities. These trends also continue in the heavier quark sets in apparent conflict with any possibility of regularity of constraints on distribution of site occupancies. (It may also be noted that the higher mass site of the strange quark set at 1/3 net charge level is co-occupied by the muon particle at a net charge level of 1 in accordance with the charge cycles of Figure 2.)

In Figure 5 the relative narrowness of the PDG mass limits for each type of quark/anti-quark set (Eidelman et al., 2004) is indicated in the logarithmic plot by the steepness of the slope of the emphasized lines connecting each set of estimated mass sites for a quark type within or near those limits. In the vicinity of the order of mass magnitude containing the pair line overlap boundary, these slopes not only reverse from positive to negative, but they also become more irregular and broken. The resulting three pseudo-quads in the higher masses are less regular than the two lower mass quad figures. The apparent irregularities appear to increase steadily with increasing line overlap in the higher mass regions.

The transition between the three lower mass quark sets and the three upper mass quark sets, which occurs in the region of significantly increasing overlap effects, is also marked by a transition in the cited PDG data between the use of <u>current</u> mass values for the lower mass quark sets and <u>running</u> mass values for the higher mass sets. (That usage is continued herein from the prior power law note.) This broad transition zone of the third quad is also a boundary between the conventional three light and three heavy quarks. The zone of transition between these various long established usages implies that there is a further significant correlation between the PDG/SM and both the continuously increasing pair line overlap effect with increasing mass in this new display format of the mass/charge power law system, and the resulting effects on the regularity of LQ mass constraints.

As a group these six very steep quark set lines (Fig. 5) tend to vary in vertical axis placement around the 1 value of *F*. With increasing mass orders of magnitude, the lines have a net trend from higher values of *F* toward lower values, somewhat consistently with the trend of the curved and dashed net charge level lines of earlier figures. (However, taken together with all the other occupied sites, there is also an apparent average trend with increasing mass of particles away from the maximum and minimum values of *F* toward the more central values. A contributory factor in this, due to the upper PDG limit of the tau neutrino mass, is the cut-off, at approximately the line overlap mass boundary in mid-graph, of the neutrinos continuation in all sites on the minimum *F* line.)

More specific factors in regularity of quark mass constraints---Behind such irregular variations as those above, which have always been problematic in any overall search for regularity of quark set characteristics, there is also a continuity in cyclic effects of the power law across the overlap transition zone which ties the disparate appearances into a common systematics. This regularizing cyclic continuity begins with the lowest mass quark sets, from which it builds up progressively.

The two lowest mass quads of Figure 5 have their vertices at the two mass value sites for each of the up (u), down (d), and strange (s) quark/anti-quark sets that were estimated by the prior note (Howard, 2005), and that are consistent with the wide PDG mass uncertainties. The only deviation of these sites from the PDG empirical particle listings (Eidelman et al., 2004) is the higher mass estimate of the power law equation for the down quark set, at 8.032 MeV. This is a comparatively negligible 0.4% beyond the PDG mass uncertainty range of 4 to 8 MeV for that particle, an uncertainty range of 100% of the lower limit or 50% of the upper limit.

The two specific estimated mass values for each of these three lowest mass quark sets have been substantiated significantly in the example appendices of the prior note cited, by demonstrating their ability as components to account systematicly for the specific empirical masses of each particle of groups of hadrons constructed from these quarks. These confirmatory examples were also effective trials of the power law equation in re-constructing and explicating the empirical mass/charge characteristics in the entire 14 particle group of the PDG neutral N nucleon Summary Table listings and in the 4 particle group of PDG Omega minus particle listings. (Eidelman et al., 2004) These 6 mass substantiations for the 3 quark sets, in the 18 baryon PDG/SM particles involved in the trials, were accomplished in the reference (Howard, 2005) in accordance both with the mass/charge power law equation (1) taken from that prior note and with the derivation of that law therein. That derivation came from composite mass relations observed between the SM quark/anti-quark sets and the hadrons composed of quarks and anti-quarks in the PDG empirical data listings cited.

There is a strong implication from these mass substantiation trials. The suitability of these three quark set estimated mass values, with demonstrated regularity, for assemblage into hadron composite particles should imply a potential full regularity of quark set values within the entire field of quantized mass option estimates, as delineated in Figure 5, even when that regularity is not necessarily immediately clear. Accordingly, some reliance is placed here on using the configuration (Fig. 5) of these three lower mass quark sets in interpreting all six.

While the mass ratios of the upper and lower values (or nominal extremes) of PDG limits of mass uncertainty for a single type of quark/anti-quark set is higher than 2.5/1 (1.5 to 4.0 MeV) at the low mass range of the up quark set, and narrows progressively through 2/1 (4 to 8 MeV) for the next higher mass down quark set to about 1.06/1 for the highest mass top quark set, these ranges are very narrow compared to the 5 orders of magnitude in mass involved in all six sets. These effectively narrow PDG mass uncertainty limits for each type of quark set (already confirmed within this power law system by correlated substantiation of the two high and low values of each of the three lowest mass sets, as just noted) are then the most stringent zonal elimination of mass value sites (or distribution constraints) for the quark sets, which are the principal occupants of sites above the mass of the tau neutrino.

There are only two other occupant particles than quark sets in this large zone of 4 orders of magnitude in mass (above the neutrinos), the closely related muon and the tau particles, to be discussed more fully after the derivation of another exponential mass relation equation. One of those, the muon, is restricted to the middle of a quark mass zone by its PDG mass limits, and can only be accommodated there in this power law system by joint occupance of a mass site with a quark set. (This is made possible by the two net charge level options of l and 1/3 on a single curved line for charged pair numbers.) The similar tau particle is near the charm quark set zone, but cannot be accommodated similarly there within the power law system because its net charge level of 1 is not compatible on a single curved line (Fig. 2) with the charm quark set's net charge level of 2/3, and is thus excluded from co-occupying a site with a charm quark.

For this power law system, the substantiation of the zonal mass site restrictions of the three lowest mass quark sets involving correlation of groups of empirical hadron masses given by the PDG/SM with hadron masses predicted by the power law, leads to a strong implication that the same correlated substantiation should potentially be derivable for the three highest mass quark sets similarly organized in other hadrons of the SM.

Accordingly, except for the tau particle site, all the large numbers of sites above the tau neutrino mass excluded by the various quark sets (i. e., sites not occupied by quarks, Fig. 5) may be considered negatively excluded from all occupancy by LQ particles. (As mentioned earlier, for pair lines above the 16(8) line, dots for these excluded/unoccupied sites of lesser importance are not placed on the pair number lines.)

The accumulated negative restrictions from occupance of mass value sites by empirical PDG particles still may well leave a relatively small number of sites unoccupied without stringent definite reason. From this point, consideration of such sites may reasonably be eliminated wherever there are no positive occupance implications through strong similarity with more definitely occupied sites or site location patterns and similar considerations of possible symmetry, or through implications for all quark sets of potential similarity (not yet demonstrated) to the previously cited examples of hadron component suitability for the two estimated mass values for each of the three up, down, and strange quark sets.

SYSTEMATIC REGULARITY OF QUARK MASS COMPARISON RATIOS---The general appearance on the graph (Fig. 5) of the quads made by connecting the estimated mass/charge sites of the quark sets would indicate that there should be some underlying numeric relation between the emphasized lines or their quad vertices. However, any such relations are complicated by the oddity that the connecting lines within the two lower mass sets slope upward steeply with increasing mass and the slopes reverse, going downward steeply, within the four higher mass sets. The two upward sloping sets are both distinctly below the 10 MeV range of the mass boundary value at which the pair number straight lines begin to overlap. The four downward sloping sets are all well above the boundary. That necessarily makes it almost inevitable systematicly that specific estimated mass sites corresponding to even a very narrow PDG quark mass uncertainty window for the four higher mass sets may be on different pair straight lines. (The cited substantiation of the two such values for the strange quark set, with the negative slope of its site connecting lines across the positive slope of the pair straight lines, even though the pair lines are relatively widely spaced here, makes this a more stringently inescapable natural occurrence.) This effect, due to the several phenomena working together, especially distorts the second quad (in the order of increasing mass) defined by the down and strange quark sets from any approximation to a parallelogram, which would be more indicative of systematicly regular mass distributions.

There are clearly some inherent displacements of immediately obvious regularity of occupied site distributions across the regular field of available sites. In any event there are two different possible measures of any distribution regularity among LQ particles in Figure 5. The first would be measurable similarities or cyclic variations of log mass increment multiples or (preferably) anti-log mass comparison ratios horizontally along the mass axis. The next would be similarities or cyclic variations, due to the charged pair ratio effect on mass, of measures taken vertically along the F axis.

In the first quad in order of mass, the parallel pair number lines of the up and down quark set values give a horizontal mass comparison ratio across the quad of a rounded value of 3.8147/1 when measured at any F value, but this is most readily measured at the maximum or minimum values from the equations (1, 2). There is only one pair number line involved here in each of those two quark sets. Direct numeric comparison with the fourth, apparently nearly similar but oppositely sloped, pseudo-quad was only possible at one F value, that of 1/2 + 1/3, since this is the only F value common to occupied sites (at 4/12 and 5/15) on both heavy lines of the fourth quad.

These two alternate optional mass value sites, correlated with the narrow PDG mass uncertainty limits (Eidelman et al., 2004), for the charm and bottom quark sets were not previously recognized as such during the preparation of the prior reference note. (Howard, 2005) Their critical locations discovered in the new graph format of this note, both in the regular system of optional mass values under the original power law and in constrained relation to the PDG mass windows, not only completed the third and fourth quads appropriately, but also subsequently provided the key to the additional regularity of distribution of the quark masses discussed in the rest of this section.

The horizontal mass comparison ratio across the fourth quad between these two newly noted points is 3.8149/1 (rounded.) This is the constant ratio (if roundings are the same) between the two <u>second</u> straight pair number lines per quark set (counted with increasing mass, or increasing charged pair steps, where there are two or more such lines) involved in the two charm and bottom quark sets, rather than between the first or last lines of each set, as with the single lines of each set in the first quad. This ratio compares well with the 3.8147/1 ratio for the first quad. The two newly noted optional mass point estimates are: (a.) For the charm quark set, 1.1628 GeV, at 4/12 on the 24(12) pair number straight line, well within PDG mass uncertainty limits of 1.15 to 1.35 GeV. (b.) For the bottom quark set, 4.436 GeV, at 5/15 on the 30(15) line, and 0.8% outside the PDG mass limits (of 4.1 to 4.4 GeV, a PDG mass uncertainty of 6.8%) (Eidelman et al., 2004).

The use, permitted by the discovery of the 4/12 and 5/15 mass option sites, of the second straight pair lines of the quark sets (where they exist) for calculating mass comparison ratios across quads, also led to the indication of hidden regularity of quark mass distributions across the second and third quads, as explained next. Use of the second pair lines may be particularly justifiable for the strange and charm quark sets since the first lines of each set are relatively only minimally involved in the range of charged pair step counts in both sets. The lowest charged pair step count within the step count range of the strange quark set is on the last and highest fully charged pair site 7/7 of the 14(7) line, which is the set's first or lowest mass line. The lowest charged pair step count within the step count range of the charm quark set is on the next to last and next to highest, almost fully charged pair site, 10/11, of the set's lowest mass 22(11) line (though not its lowest mass occupied site which is at site 4/12 due to line overlap effects.) Thus only one to two steps of each of the first lines is occupied or in the step sequence between sites occupied by either set. This is much less than the four to five steps in those sequences on the second lines of each set. Consequently, adopting the rule of using the set's second pair line, where there is one, for comparisons in mass ratio across the quads effectively weights the ratios to emphasize the importance of the lines with the greatest proportion of the charged pair steps between the sites of the lowest to highest step counts for the quark set. As will be seen, this brings order of additional mass distribution regularity under the power law system to the empirical PDG mass uncertainty constraints for the quark sets. This additional constrained regularity under the power law is established through the mass comparison ratios across all the quark sets. (This is the first indication of the increasing importance of the charged pair step counts over the following sections. It is not an isolated factor here.)

The second line rule brings the third quad (or pseudo-quad) in mass order to a closely harmonic relation in horizontal mass comparison ratio. The third quad resolves, on the second pair lines of the strange and charm quark sets involved, at a rounded 11.39/1 ratio measured at any horizontal section of the lines (preferably at the points of maximum or minimum *F* where precise mass site values are readily available.) This ratio is quite similar to 3 times the first quad ratio of 3.8147/1 or 11.444/1, with a deviation of 0.47%.

The second quad in mass order contains the distortion due to bridging from the zone of positive quark set line slopes with the down quark set to the zone of negative slopes with the strange quark set. It is thus also the area of change from quark sets on a single pair line to two line quark sets. To bridge this gap reasonably, the second line rule might be adapted toward a line and a half intermediate at which two definite points of measurement can be specified. Since the higher down quark mass value (of the estimated masses involved here) is observably near the perpendicular bisector of the strange quark set mass line on the log plotted graph, the best available definite approximation of a suitable measurement point across the quad from that down quark value, and on a line and half intermediate between the first and second strange quark set lines, would be the antilog of the mean log of the two strange set mass estimates of 82.467 MeV and 107.19 MeV. Since the down quark site is actually slightly below the perpendicular bisector line noted, and since the two measurement points cannot then be on the same *F* value, the resultant approximate measurement must necessarily be slightly high. The rounded value of the noted antilog is the same as the geometric mean of the two strange quark set mass values, or 94.019 MeV. The horizontal mass comparison ratio between this and the higher down quark mass value of 8.03183 MeV is 11.7058/1. This ratio is 2.28 % higher than the 11.445/1 multiple by 3 of the 3.815/1 ratio for the first undistorted quad.

In view of the known approximations just noted, this deviation for the second quad, and also the much smaller deviation for the third quad, are small enough to be reasonably accepted as an approximate equivalence of values. If so, then there is a clear, though very generalized, cyclic or harmonicly related series regularity of quark set mass comparison ratios of 1, 3, 3, 1 over the first five sets of quarks in the first four quads in order of increasing mass.

(A certain amount of the deviations from perfect cycles comes from effects of line overlap. In the bottom quark set the line to line mass advance ratio is only 1.513 or 1.473, while the advance ratio along each line is exactly 3. About half of each line is overlapped. The masses appear to be cyclicly doubling back on themselves with increasing pair numbers. Where possible, taking comparison ratios only at a fixed *F* value would obviate this source of apparent deviations in the higher mass ranges.)

The mass comparison ratio across the fifth (pseudo-)quad between the bottom and top quark sets may also be calculated by the second line rule with well occupied pair lines. The resulting ratio of 34.014/1 is slightly below 9 times the first quad ratio of 3.815/1 or 34.335/1, with a deviation of 0.94 %. That would bring the overall series to 1, 3, 3, 1, 9. This may reasonably be considered a suitable equivalence of a regular cyclic series of harmonic ratios between mass point values constrained in distribution by quantized numerics working under the power law across the offsetting displacements that increase regularly above the line overlap mass boundary.

The sequence of net charge levels with increasing mass in the quark sets also has a distinct cycle like both the mass comparison ratio cycle and the cycle of vertical placement of the quad vertices on the *F* axis scale. This sequence is: Up quark set, 2/3. Down, 1/3. Strange, 1/3. Charm, 2/3. Bottom, 1/3. Top, 2/3. In simpler ratios it is 2, 1, 1--- 2, 1, 2. With the indicated divided phrasing, this correlates exactly with the sequence of the quads in separating the charm quark set from the strange quark set in different particle classification zones and in matching the 2, 1 sequence to the first and fourth quads with similar shapes (Fig. 5) and with the same mass comparison ratio noted earlier. The next following quad in each case (second and fifth) has a reversed positive slope of the lines connecting sets and a harmonic multiple (times 3 or 9) of the minimal mass comparison ratio as before. The divided phrasing corresponds with the transition distortions due to line overlap in the third quad. The quad mass comparison ratio series may also be phrased as 1, 3---3---1, 9 for the three particle classification zones with the third quad in the center both separating and connecting the up/down/strange quark set group and the charm/bottom/top quark set group. In doing so the third quad drops the initial higher vertex of the fourth quad from the fully charged pair line by one step, keeping it at the same net charge level of 2/3 as the equivalent vertex in the third classification zone of the first quad in the first classification zone. It could not do that on the fully charged line without advancing or receding by one pair number line and so disrupting the harmonicly related mass comparison ratio sequence in its cyclic series. The first two quark sets were noted earlier to be positively constrained in specific lowest mass sites (for their charges) in the graph of mass distributions. By cyclic sequences from the first two sets, the rest of the quark sets and their quads are constrained to other specific sites of combined mass and charge organized in a particle classification system of a low to minimal mass zone of simpler particles with no overlap of pair number lines, which is gradually phased from the overlap boundary into a sparsely occupied central transitional zone of slowly increasing line overlap, and a high mass zone of unstable particles with more sites in complex mass/charge sequences due to overlap at $p/n \geq 1/3$.

The quark set mass distribution now found to be regularly cyclic under the mass/charge sub-structure power law from the prior note (Howard, 2005) (in addition to the regularity of the optional site mass values themselves) was originally distributed over the alternative optional sites in correlation with the PDG data lists. Thus, this harmonic cyclicity of mass comparison ratios from quark set to quark set with increasing mass constitutes a further form of correlation of the power law system with the PDG empirical mass values for these quark sets. (Eidelman et al., 2004)

From the substantiation of the estimated mass distributions plotted in the quads of the up, down, and strange quark sets, in the trial examples of the prior power law note (Howard, 2005) which confirmed their suitability under that note's equation (1) for applications in hadrons, there is an implication that still further explanation and substantiation or reductions in numbers of the present estimated mass distribution sites of the charm, bottom, and top quark sets might be found in similar trial examples of suitability in hadrons. The iterative trials for these three quark sets would be more extensive than the prior example trials due to the additional numbers in each set of optional mass values within or near the PDG limits. (Eidelman et al., 2004) The trials have not been done nor definitely scheduled to date of writing. (The first step would be to select for the trials sets of empirical hadron particles with all the necessary information, for elimination of ambiguities and uncertainties, fully accredited by the PDG by inclusion in the final Particle Summary Tables, or filled in by very close equivalents in less highly accredited PDG Particle Listings and validation rating notes if unavoidably necessary, as with the Omega minus particle examples for the strange quarks previously cited. Among the multiplicities and uncertainties of particle proliferation, the first necessity is for the mass/charge power law system to match the most definitely certifiable and repeatedly confirmed and complete empirical data in hadron composite structure, with emphasis on the least exotic and lower mass levels.)

There are several points to note in such trials. Where the three lower mass quark sets have only two estimated (and substantiated) optional mass values per set, for the three higher mass sets, correlations with the PDG mass uncertainties for each set now make it necessary to include three sites in the charm set, four sites in the bottom set, and four sites in the top set. Two of the optional estimates for the charm set are very close together at 1.1628 and 1.1665 GeV on the 4/12 and 10/11 sites respectively (Fig. 3), reversing the mass order in the pair number line order at the correct net charge level of 2/3. The third charm set estimate is the one previously noted as farthest (of all power law mass estimates) outside the empirical PDG limits (here 1.15 to 1.35 GeV, a 15 to 17% range), by 3.3% high, at 1.395 GeV on site 6/12 (Fig. 3). The deviation of 3.3% is close enough for correlation as an optional estimated mass alternative, but this site may, or may not, be found unnecessary in substantiation trials.

All four of the bottom quark set's estimated mass options have the correct net charge level of 1/3, but are slightly outside the PDG mass limits of 4.1 to 4.4 GeV, a 6.8% uncertainty range. As cited in the prior reference note, the lowest mass estimate for the bottom set is 1.9% low at 4.0218 GeV on site 9/14 in the graphs, and the least deviant high estimate is 0.23% high at 4.4106 GeV on site 1/16. The two newly noted bottom set estimates are sited on pair lines out of mass order and are 2.82% high at 4.524 GeV on site 11/14, and 0.841% high at 4.437 GeV on site 5/15. New empirical data may lead to changes of the PDG accredited limits of mass uncertainty from time to time, and these only available options are not considered to be excessively deviant for alternative optional correlation with the empirical mass constraints. The principal concern here is the question of how many of these optional mass estimate sites will subsequently be found by trial in calculation experiments under the power law to be suitable for application in some part of the wide range of composite hadrons, whether it will be only two or three or all four of the available options. In the meantime each of these optional values must be weighted about 50% tentative by implications of limited similarity with the three lowest mass quark sets which have only two mass options each. This might particularly apply to the sites that most disturb the regularity of the quad patterns themselves, until adequate hadron trials become decisive. Site 11/14 is clearly out of regular order at one step up from the upper charm site's net charge level line (Fig. 2). It may be in excess. (Or it may develop that the hadron suitability criterion should be reversed. It may be that because of a marginal co-location of a regularly possible alternative site such as bottom site 11/14 just outside of a normal empirical mass uncertainty for a quark set, a suitable empirical hadron will occur on occasion in the natural proliferation of particles. All the potentially possible, marginal combinations might be found to occur at low probabilities and low rates depending on the numerical distance of an optional quark mass from the most probable empirical value. To eliminate or establish such a type of possibility, it might develop that large numbers of substantiation trials over many hadron groups would be required.)

Similar questions apply to the three charm set options above, and to the four top quark set options. All of these top set options at the correct net charge level of 2/3 are well within the PDG mass uncertainty limits of 174.3 ± 5.1 GeV (Eidelman et al., 2004). With the sequence of pair line listings again somewhat out of mass order, the top set options are 172.12 GeV at site 18/26 (Fig. 5), 170.99 GeV at site 12/27, 176.94 GeV at site 8/28, and 177.32 at site 4/29. The last two are newly noted here, and were not recognized in the first power law paper.

In the discussions of vertical placement of quad vertices in the next section, implications arise that a fifth site option in the top set would make the quad mass distributions more regular. This would be a new lowest top quark set option 1.556 % below the PDG uncertainty limits at 166.568 GeV on site 24/25, as indicated on the graph (Fig. 5) by the dashed emphasized line at high *F* value. (If that mass value option should fall within changes in the PDG Summary Tables, or become substantiated in hadron structure trials, the present first pair number line of the top quark set through site 18/26 would become the well occupied second top set pair line. Then, for the quad harmonic cycle, the mass comparison ratio above the bottom quark set second line for the fifth quad would become 27.1215, with a deviation of 1.54 % from the 7th multiple of the first quad ratio (of 3.815/1) at 26.705. The overall mass comparison ratio cycle series for the five quads of Figure 5 would become 1, 3, 3, 1, 7, which, as a series of small integral prime numbers, would have approximately the same level of cyclic regularity as the present series of 1, 3, 3, 1, 9. There is also an implication of closer regularity in that the mass estimate option on the more heavily weighted, second line of the set would then be the present estimate option of 172.12 GeV, which is closest to the present biennial PDG central empirical value of 174.3 ± 5.1 GeV for the top quark set.)

Quark mass/charge regularities by relative *F* factor---As noted earlier, the quark set mass lines and their quad vertices in Figure 5 are sensitive to the *F* factor in the mass/charge power law equation (1, 2). This sensitivity is depicted by the dashed and curved net charge level lines which alternate across Figure 2. The upper vertex of the emphasized line of estimated masses for the up quark set is at the fully charged end of a pair line and on a 0 or 2/3 net charge level line. The lower vertex for the set on the same pair number straight line is necessarily 2 net charge lines below the upper because of the charge line alternations. It cannot be 4 charge lines lower, since that line is at 0 charged pairs only (the zero axis for charged pairs.) Neither can a quark set with two mass options (as substantiated in the prior reference note to suit hadron mass windows) and a 2/3 net charge level occur at one or at two pair lines lower mass in this non-overlapped region of the graph for the same kinds of reasons (though there is a single site for 2/3 net charge at two lines lower.) The lowest mass quark set is thus at the lowest mass level, with the fewest components, that can provide its quark types of features in the mass/charge power law system. Empiricly, low mass simplicity appears to be associated in all types of particle observations with high comparative numbers present in ordinary stable matter. (This implication of stability associated with small simplicity has similarity to minimum prerequisites or assumptions in establishing strong theories.) The up quark set must necessarily, then, have its upper vertex on the fully charged pairs *F* value line at the next higher fully charged mass site above the electron, in order to allow a similar 2/3 net charge site two steps below it. The set also has this least number (two, not four) of charged pair change steps between its mass sites of equal net charge level as an additional aspect of being at the highest overall *F.* From this baseline group of definite constraints to the highest *F* values for the most simple site locations (in terms of both minimum number of power law components and correlation with lowest empirical mass) for the probably most numerous quark (from its uud predominance in the proton of hydrogen), the other quark set distribution locations proceed with increasing mass in the characteristic pattern of the quads.

The quark set most closely related to the up quark set is the down quark set on the opposite side of the first quad (Fig 3.) Relative to the vertices of the up quark set, the upper and lower vertices of the down quark set are displaced downward on the graph by one curved charge line to the 1/3 net charge level sites (as required for empirical hadron balancing) at lowest available mass on the next higher pair number line. Here, for the least, and therefore simplest, mass the set is not forced to the highest available *F* site on the pair line. These characteristics allow the down quark set no other mass/charge site locations that are as positively suitable for participation in low mass stable hadrons. That trend in nature for the quantity predominance in stability of the small and simple, sets the cyclic pattern for the mass distributions of the quark sets in the quads. This appears clear. Furthermore, the down set can not take the alternative option of moving up to the empty higher mass site 5/5 for fully charged pairs of the same 1/3 net charge level on the same pair number line. The 3/5 site mass of 8.032 MeV is itself 0.4% above the empirical mass limits of 4 to 8 MeV. (Hadron balancing regularities which may underly the PDG mass limits for quarks may outweigh therein the matter of minimal mass sites discussed above for the down quark set. Still, the minimal mass site characteristic is present in both the empirical data and in this system of constraints on mass distribution for particles under the power law.) In addition, the vertical placement pattern for the higher mass quark set in this first quad is repeated in the fourth quad in step with the mass comparison ratio pattern repetition. Though somewhat tentative (as noted), the fourth quad, as drawn, shows the upper line from the charm quark set going to the bottom quark set value at 9/14 that is one net charge line down from the charm site (similarly to the first quad), not to the less regularly cyclic bottom quark site at 11/14 which is one net charge line higher than the charm site, and therefore higher in mass than the 9/14 site, as well as not similar to the first quad pattern.

The sequence of net charge levels with increasing mass in the PDG quark sets has a distinct relation to both the mass comparison ratio cycle and the vertical placement of the quad vertices on the F axis scale. This sequence undoubtably is also strongly influenced by hidden regularities (beyond this scope) in hadron balancing, which must be matched herein through the PDG empirical mass/charge limits. The net charge level sequence is: Up quark set, 2/3. Down, 1/3. Strange, 1/3. Charm, 2/3. Bottom, 1/3. Top, 2/3. In simpler ratios it is 2, 1, 1--- 2, 1, 2, with the indicated divided phrasing into a cyclic sequence that has further implications. The natural sequence grouping here (based in the cyclicly recurrent emphasis on the consecutive 2, 1) correlates exactly with the previously established sequence of the quads. (This would separate the charm quark set from the strange quark set, with which it is conventionally grouped by LQ particle generations.) The 2, 1 sequence here matches the first and fourth quads with similar shapes (Fig. 5) due to a similar negative slope of the lines connecting sets and the same minimal mass comparison ratio noted earlier. The next following quad in each case (second and fifth) has a reversed positive slope of the lines connecting sets and a harmonic multiple (times 3 or 9) of the minimal mass comparison ratio as before. The separation between the two groups in the net charge level sequence of quark sets noted above corresponds with the linkage of upper and lower connecting sides of the first quad in mass order that is entirely above the transition distortions involved with the beginning of pair number line overlaps, that is, the third quad. In the quad mass comparison ratio series.(as now adapted) 1, 3---3---1, 9 that quad is the central quad shown as an ungrouped item. (This separation of the third quad would tend to reconnect the strange and charm sets as a separate group similar to the conventional second generation. But that reconnection is misleading. The real implication of the quad structure here is to demonstrate around the first quad the major natural grouping of the first and second quark sets, the up and down, and a similar grouping around the fourth quad of the 4th and

5th sets, the charm and bottom. With the location of its lower mass site, the mass distribution of the third strange quark set implies completing the second quad as a rounding off of the major first quad group while also distorting the second quad in continuity into the third quad with the location of the higher mass site for the strange set on the next higher pair line. Like its name, the quad position of the strange quark set implies an internal duality of functions, since it also provides with the third quad, of which it is a part, an overlap-transitional group between the other quads and quark sets.

The third quad space on the graph (Fig. 5) does then appear both to separate and connect the up/down/strange quark set group and the charm/bottom/top quark set group and also to separate and connect the first/second quad group and the fourth/fifth quad group. In the process the third quad further develops the overlap-related transitional features introduced by the strange quark element of the second quad. In the mass distributions the third quad drops the initial higher vertex of the fourth quad from the fully charged pair line by one step, keeping it at the same net charge level of 2/3 as the equivalent vertex of the first quad. It could not do that on the fully charged line without advancing or receding by one pair number line and so disrupting the harmonicly related mass comparison ratio sequence in its cyclic series. This outcome emphasizes the interlocking nature of the quantally estimated mass/charge site distributions as constrained by the PDG empirical mass limits and charges related to composite hadron particles. The first two quark sets were noted earlier to be positively constrained in place in the graph of mass distributions , and by cyclic constraint sequences from the first two, so are the rest of the quark sets and their quads.

The entire quad system as it stands, in closest available correlation with the PDG empirical mass limits on quark set masses, is not fully regular, even in its own peculiar way, especially in one particular aspect. The third (strange) quark set returns the second quad at its higher vertex to the same fully charged pairs step level as that on which the first quad higher vertex began. This might imply that the sixth (top) quark set should in systematic regularity return the higher vertex of the fifth quad to the same level of a single step short of fully charged as that on which the fourth quad higher vertex began. As previously indicated, if the PDG lower mass limit for the top quark set were reduced by 1.556% (or disregarded for this purpose), there is an appropriate mass option of 166.518 GeV at the 24/25 site, as indicated by the dashed line above the fifth quad. This would make the entire quad system of quark set mass site estimates a significantly more regular numeric distribution. Whether such a top mass value would be suitable for composite structure in hadron classes of particles is thus implied here to have a potentially positive answer, provided that there is given a sufficient, fully accredited, empirical data base for a hadron mass substantiation procedure similar to the one in the first power law note to test that potential.

As a further note on the quark set quads, the pairs of up and down quark estimated sites (Figs. 2, 4) are separated vertically by the spaces of 2 curved net charge level lines. In the strange set, the vertical separation is by 4 such line spaces. (It cannot occur at 2 such line spaces, since either the upper mass of the set would be too high or the lower mass of the set would be too low, well beyond PDG uncertainty limits. Also the quad structure would be severely distorted out of any constrained regularity of mass distribution.) In the charm set, this occurs by 4 and then 2 line spaces for a total of 6. In the bottom set, by 4 and then another 4 line spaces for a total of 8. In the top set, the total number of such curved line spaces, if plotted, would be a surprisingly large 14 lines. This jump from 8 to 14 lines might imply by quad pattern similarity that one or both of the two lowest *F* and highest mass sites for the top quark set at 8/28 and 4/29, though well within the present PDG mass uncertainty limits as noted earlier, may be in excess of the requirements for hadron construction.

As noted earlier, the highest mass bottom set site (Figs. 2, 5) at 11/14 is clearly two net charge curved lines too high for the cyclic mass distribution pattern established for the first and fourth quads by the higher of the two down quark set vertices, which is one curved net charge line space lower than the higher up quark set vertex. Contrary to that cyclic relation, the 11/14 site is one curved net charge line higher than its reference at the 10/11 site of the charm quark set. Consequently, the quads as shown (Fig. 5) assume the implication that the 11/14 site is in excess, and it is shown only as a dashed spike. The 9/14 vertex indicated below it does follow the regularly cyclic mass distribution pattern of being one curved line down from the initial vertex for the fourth quad. (The line overlap boundary dislocation or offset effect may possibly be considered to account for other lesser differences in the quads. However, that effect did not prevent the lowest *F* vertices of the first and fourth quads from occurring on the same curved net charge level line at sites 1/5 and 1/16 (Fig. 2) in step with the 2,1 charge cycle groupings for those quads noted earlier.)

Derivation of a new exponential law for LQ mass distributions---The steps of charged pair change along the pair number lines (Figs. 1, 2) from the mass/charge power law equation (1) may be summed (in step-by-step count as noted earlier) to derive an exponential law for the regularity of the masses of the LQ particles in the mass sites of the present graph (Figs. 1, 2) as a function of a slightly modified, typical series sum equation for an equivalent to the step count. As will be seen, this law is distinctly different from the original mass/charge sub-structure power law in its way of computing mass values, and it involves a much smaller variable exponent. While the new law computes mass values by a separate method, it is not independent in derivation. Consequently, it can provide essentially a co-active verification of the constrained regularity of mass distribution of the LQ particles across the U range of the power law, rather than an independent substantiation of the derivation of the power law and its absolute mass estimates for particles. (That further substantiation will have to come from the longer term and broader series of trials in accounting for groups of hadron masses in the proliferation of particles by assembly from estimated quark masses under the power law, as previously noted.)

The series equation base (3) of the new exponential equations (4, 5) is most readily understood by setting it up as a condensed equivalent to the mass site step process previously described in detail. The steps begin from a 0/0 zero position with 1 step to the 0/1 site on the first pair line, followed by step 2 to the 1/1 site, and by a flyback step 3 to the 0/2 site on the second pair line. To account for the initial step and the summed flyback steps, which are not in the usual series, the equivalent steps *p* of charged pairs in the final pair line (as in equations 1 and 2) must be added to the summation of the conventional series through the final nth number of the final pair line as if that line's quantity were completely included (per the usual summation without a fractional inclusion by means of *p*, which cannot exceed its line number.) This small adjustment of the usual series summation obtains the total sum *S* at a given site of the individual steps *s* to that specific site on a pair line; not just to the end of the prior line. Accordingly,

$$S = \sum_{s=1}^{n} s + p = \frac{n\,(n+1)}{2} + p = \frac{n}{2}\ \{n + 1 + 2\frac{p}{n}\}, \qquad (3)$$

which accurately counts the total steps to any mass value site corresponding to a mass defined by equation (1) as graphed in the figures (Figs. 1, 2). For example, the hand-counted and the computed number of steps to the higher mass down quark site 3/5 is 18.

If the masses at the sites for $p = 0$, $0.5n$, and n (where $F = 0,5$, 1.0, and 1.5), plus all occupied sites not in these three groups, are graphed as a function of S on a very large scale plot on log vs log paper (not shown), it is apparent that the plot starts at low values with small triangular waves, of which one corresponds with each pair number line (Fig.1). These waves oscillate around a smooth line which may be drawn in with an exponential slope near 3 through the central (numerical mean) points equivalent to $F = 1$ (Fig. 1). On such a (not shown) very large log plot the wave amplitude shrinks in apparent scale with increasing S and mass to a very small minimum size near $S = 21$, where the mass values pass the 10.9 MeV area of the line overlap boundary (Fig. 1) with the first overlap of 0.05 MeV in mass. Just below that point the triangular waves around the central mean line invert their phase. Above that point the waves steepen into obvious sawteeth shapes during flyback and grow again in apparent mass amplitude around the mean line with increasing S, even on the logarithmic scale. The slope of the central mean line around which the waves oscillate is not constant, but decreases in a very shallow curve and approaches 3 asymptoticly with increasing S (through 439, the highest occupied site value involved in the top quark set area, or through 465 to complete that sawtooth wave cycle.) The central numeric mean line is singly curved concave downward. (If the starting step is cut by plotting against $S - 1$, the central line on the very large plot appears to straighten at the lower range to a constant slope of 3. But it is not straight. It has a smaller double or reversing curvature. No equation was derived for this type of step count minus one.) Due to the waves/sawteeth in the complete (not shown) curve, the slopes of lines from occupied site to occupied site of any type may have a wide variety of values, ranging from high negative to high positive values, depending on the number of incremental steps between the sites as well as their exact locations on the curve. The very high local negative slopes along sawtooth flyback lines in the high mass ranges have no apparent further application. Between typical pairs of occupied sites, except for pairs in which both members are above 1 GeV and are also immediately adjacent (in a single quark set or on the same triangular wave), slopes are usually in the general order of positive 3, like those of the central mean curve.

From that general exponential slope, it is clear that the masses of sites and particles on the complete curve must equal some function near the cube of the summed number of steps. Trials suggest

$$m_p = \frac{2m_u}{3} S^3 \times S^y \tag{4}$$

where symbols continue from the prior equations (1, 2, 3) except for an unknown *y*. If 2n is substituted for N in equation (1), and that is equated to (4) with substitution from (3), the combined equations can be solved for

$$S^y = \frac{2^8 \; n^3 \, (1 + 2\, p/n)}{(n + 1 + 2\, p/n)^3} \tag{5}$$

and $y = \log S^y / \log S$, if needed.

Thus, mixed exponential/power equations (4, 5), in terms of summed steps in the changing of the numbers of charged pairs of sub-structural components in LQ particles over their entire U range, are an alternative to mass/charge power law equations (1, 2) for investigating the systematic regularity of the mass distribution relations of these particles, especially under constraints from the interpretation of Figure 5. One lepton group remains to be examined in this way (in addition to comparing it with the prior groups.)

THE THREE UNIT-CHARGED LEPTONS (UCL)--- The UCL estimated mass sites in Figure 6 are indicated by the three larger circles connected by emphasized lines. Their location with respect to the quads of the quark sets are indicated by showing the quads in the background with dashed lines. The general shape of the two heavy connecting lines for the UCL group is somewhat similar to the pattern of the top lines of the first two quads of the quark sets, though broadened.

The particles of the UCL group, the electron, the muon, and the tau particle, and their anti-particles, are all at the unit net charge level of 1, whether minus 1 in the particles, or plus 1 in the anti-particles. The charge of the electron conventionally defines the unit charge, and the power law system equations (1,2), as well as the new law above (3, 4, 5), conform to that rule. Other than the conventionally composite proton (and anti-proton, both baryonic hadrons), these three UCLs might be considered the principal sub-atomic particles of that net charge level, with primary emphasis on the electron. The UCL empirical masses are all listed by the PDG to six to eight significant figures (Eidelman et al., 2004). However, the muon and the tau are quite rare and unstable compared to the ubiquitous electron. Their mean PDG empirical lifetimes are well beyond 30 and more orders of magnitude shorter than that of the electron.

In the power law system the UCL mass sites can only fall on the 1/3, 1 net charge level curved lines (Fig. 2), which constrains them by 50% within the quarter of the graph including and above the curved line the electron is on. In fact the muon is at the 3/8 site on that curved line for the same number ($p = 3$) of charged pair steps as the electron. The tau is six curved lines higher at the 9/12 site.

The electron is located at site 3/3 on the line of fully charged pairs (F=1.5) at the beginning in Figure 2 of the first curved and medium dashed net charge level line for charges of 1/3 or 1. This particle has the minimum number of pairs that could give a net charge level of 1, and there are no extra pairs. Therefore, it is located at the distinctly lowest mass (by half an order of magnitude) which could in this mass/charge power law system have a net charge of 1. It is positively and fully constrained to the one optimum, or lowest mass, site in the LQ mass distribution for quasi-unlimited stability of a unit charged particle. There are no PDG empirical LQ particles of similar net charge level (Eidelman et al., 2004), nor are there occupied sites of that level in the figure graphs (Figs.2, 6) within about two orders of magnitude in mass, though there are in that range eight optional alternate sites in this power law system of intermediate potential stability likelihood for unit-charged particles (where such particles might possibly be implied to have existed at one time or another, or to be still extant at some location in nature.) Since the listed mass estimates of the mass/charge power law equation (Howard, 2005) were calibrated by using the electron as the reference mass to within 0.01% (including rounding) of the PDG empirical electron mass value, the estimated mass of 0.511 MeV for the electron is considered to have no deviation for the purposes of these notes.

The estimated mass value of 107.2 MeV for the muon in the power law system has a deviation from the PDG empirical value of about 1.5% high, some part of which may be due to roundings in this and the prior note. The estimated mass of 1. 744 GeV for the tau particle has a similar deviation of about 1.9% low. These opposite deviations may imply that the power law system field of mass/charge site options specified by equations (1, 2) for the LQ particles (Figs. 1, 2), needs an additional, cyclic correction factor in mass distributions, at least for UCLs above the pair line overlap boundary. A portion of such correction may be provided by the new equations (3, 4, 5) of mass as a function of charged pair step counts.

The step number relations of the UCLs are indicative of regularity of masses in an additional way. Their masses extend over a mass range in which the total additive exponent of S in equation (4) ranges from just above 4 to just above 3. The electron is at step $S = 9$, with a step increment above the first step of $S - 1 = 8$. The muon is at step 39, with a similar increment of 38. The tau is at 87, with a similar increment of 86. If the ratio of these $S - 1$ increments for any increasing combination of two of these particles is raised by a constant power of 3.42554, this is equal to the ratio of the estimated masses of the same particles, within 0.86%. The exponential ratio is equal to the mass ratio within 0.001% for the longer range from the electron to the tau. It is higher than the mass ratio for the electron to muon range, and about equally lower for the muon to tau range, both within 0.86%. (Taken with the empirical mass deviation noted , this near correlation might perhaps be taken to have a further implication. Since their empirical lifetimes are of the order of micro-seconds or less, the muon and tau particles originate in the channels of their generation within the times of their observations. In conjunction with the deviations between their empirical masses and the power law estimates, and with the smaller deviations of their mass ratios in the exponential law, the brevity of the empirical observation period might imply a potential possibility for brief post-generation excitation mass increments for the empirical muon and tau, which effect would be beyond the scope of the power law.)

Similar step calculations between sets for the masses in the highest equivalent F locations in the quads (or separately for the masses in the lowest locations) of the quark sets, as well as calculations for neutrinos, have similar types of outcomes in mass ratios versus step counts, but with other exponents. However, the percent deviations between some sets, depending on the exact procedure followed in calculating, may be as much as three times higher than with the UCLs. Even assuming that all deviations are as large as 2.5%, that is much less than the percent difference between optional unoccupied (or occupied) mass steps along a single pair number line for all cases up to the top quark set, where the step separation is still about 7%. The constraints of the step process then are sufficient to prevent confusion of quantized mass/charge site designations for particles from site to site, even if adjacent sites were interchangeable, which they are not because of the 50% distribution constraints defined by the curved net charge level lines (Fig. 2), as noted earlier. Mass percent differences along the interchangeable charge curved lines would be similarly distinguishable, or more so.

The broadened similarity of the continuous line of the UCL mass distribution V pattern to the V pattern repeated in the heavy dashed lines of the quark set quads, as well as in the heavy dotted principal lines of the neutrinos (Fig. 7), implies strongly that similar interactions within the three different types of mass/charge relations constrain the cyclic regularity of mass distributions of all the LQ particles to their very limited number of specific sites. The interaction that defines the V on the graph has the form of a decrease in F (where it is not already at the minimum) with increasing mass, followed by an increase in F with increasing mass in each kind of particle. In the quark sets the cycle is repeated. This cyclicly repeated distribution of masses illustrates the analysis in the text of the relations between quantized mass/charge components in composite LQ structures, with conserved charge, as described by the sub-structural mass/charge power law and the co-active exponential law, which define and control the computed masses of these conventionally elementary particles, in correlation with the empiricly observed masses.

In this light, the charge step count relations themselves, as defined by equations (4, 5), must have a major controlling effect on the mass distributions within and between the separate types of particles (UCLs, quark sets, and neutrinos) in a systematicly regular way that appears in the three sets of bold lines of Figure 7. The UCL pattern, the quark set quad pattern, and the neutrino site pattern of estimated and correlated mass distributions for each type of particle are internally controlled similarly into cyclicly similar V formations in the mass/charge relation space of the graph. Thus, the distribution of estimated LQ particle masses in the primary usual U range of the power law system is systematicly regular and fully constrained to their specific and uniquely quantized sites (Fig. 7) by the interaction of present equations (1, 2, 3, 4, 5) coupled with the substantiating hadron composite relation of equation (1) of the prior note (Howard, 2005). This is most positively true of the distribution of masses in the first two quads of the first three quark sets in order of mass which have already been substantiated by composition of hadrons in that prior note. The PDG particle mass uncertainty limits (Eidelman et al., 2004) supply further permissive confirmation of the unique mass constraints of these co-active laws.

(This result might also imply that there may be, within the step count numerics of the power law/exponential law field of optional mass/charge value sites, a still hidden, but more exact, type of comparative cross-check relation between mass sites, than that of the preceding paragraphs discussing ratios, which will indicate the precision of quantal mass distributions under the co-active mass/charge power/exponential law equations with even smaller percent deviation.)

As a further possible implication, while the electron is on the next pair number line below the up quark set, the muon jointly occupies the same mass site as the higher mass option of the strange quark set, and the tau is on the same pair number straight line as the highest and lowest mass options of the charm quark set (or one line above the intermediate mass option of the charm set.) If this pattern were taken to indicate a regularity of site relations between the UCLs and the quark sets or their quads, that would imply that a fourth UCL might potentially co-occupy one of the heavier top quark set options or be near one of them in *F* value at one pair number line higher or in the same pair number line. It may also become interesting (or perhaps not necessarily coincidental) at some point that empty optional mass/charge site 7/11 (not marked on graphs), on a proper net charge level line for UCLs (net charge level of 1), has an estimated mass within 0.264% of the PDG (Eidelman et al., 2004) empirical mass of the proton. (Due to incompatible charge differences, this could not have any relation with the empirical neutron mass, which is also within the same percentage range. Because of the extremely short lifetimes of most unstable LQ particles within two orders of magnitude of the 7/11 site mass near 1 GeV, there is no implication of significance in connection with the long-lasting proton.)

Screening for electron neutrino mass options---The PDG upper limit of mass for the tau neutrino of <18.2 MeV (Eidelman et al., 2004) is very close to the estimated mass of 18.169 MeV (Howard, 2005), and both are very close to two orders of magnitude below the mass of the tau itself at 1.777 GeV (PDG) and 1.744 GeV (estimated.) A similar relation is present between the mass of the muon neutrino at <0.19 MeV (PDG) and 0.1703 (estimated) with both about three orders of magnitude below the mass of the muon itself at 105.7 MeV (PDG) and 107.2 MeV (estimated.) This, taken together with the prior note's implications from the three divergent groups of empirical neutrino mass findings in the current literature cited there, would seem to imply that there might be a basic upper limit electron neutrino mass option more similarly located in the usual (U) range of the power law. The PDG upper limit of <3 eV is five orders of magnitude below the 0.511 Mev of the electron mass. The U neutral mass option of 234.13 eV at site 0/1 would be more similar to the two to three orders of magnitude below the primary UCL of the other two cases, and would complete the PDG family of upper neutrino mass limits within the U range of the power law system.

However, this reasoning may be misleading. If the minimum *F* axial line of Figure 7 were extended to include the PDG electron neutrino upper limit of <3 eV in the power law's extreme E range and graph lines were drawn on it connecting each UCL primary with its conventionally related neutrino, it would be immediately obvious that the slopes of these lines would be very similar. Calculations of the slopes would confirm that. Furthermore, checking the lengths of the connecting lines in arbitrary units would show that, with about 4% deviation, the line lengths vary inversely as the 2.075 power of the log of the mass of the primary UCL in electron-Volts. This tends to confirm within the power law system a numerical regularity of the present PDG mass limits for the three primary neutrinos.

If a line is drawn from the electron to the optional neutrino mass site 0/1 for 234.13 eV, those slope and line length evidences of regularity with the empirical tau neutrino and muon neutrino mass limits are broken. To regain that form of regularity, each of the two heavier neutrino mass limits would have to become enough higher empiricly to permit assignment of the next higher alternative optional mass site on the same 0 net charge level curves. In that doubtful event, the redrawn line slopes from each UCL particle site to its designated neutrino site would again be almost as similar to each other, and the line lengths in arbitrary units, with about 2% deviation, would vary inversely as the 1.75 power of the log of the UCL particle mass in electron-Volts. There are no other contingently regular power law alternatives within the U range to the present very regular constraints on primary neutrino masses (i.e., those at or near the PDG upper mass limit for each designated type of neutrino.)

Conclusion---The masses of each specific type of LQ particle, previously estimated as quantized optional mass values in systematic regularity under the mass/charge sub-structure power law equation in a prior research note (Howard, 2005), have now been newly and positively constrained to their uniquely specific quantized mass values which generally correlate with the PDG (Eidelman et al., 2004) empirical particle listings. The new constraints sufficiently define the quantal, systematicly regular numerical distributions of estimated masses between all the individual LQ particles and the types of these particles. There are a few implications of potential small adjustments in empirical listings, most notably that there may be two mass values in each quark set. By means of these new confirmations of systematic regularity of LQ particle masses under the power law, this note now fully confirms the findings of the prior precedent research note with respect to correlation of these LQ particle mass estimates with structural mass requirements of PDG empirical hadron particle data groups using a composite exponential law of quark and hadron mass relations in equation (1) of the prior note, from which the power law was derived. These new findings are supported by deriving from the power law equation a new exponential law which re-interprets it and acts jointly with it. A new and possibly more suitable grouping of the LQ particles for analysis than the conventional three "generations" is indicated. As a more general new result, the mass and conserved charge properties of the entire empirically observed and PDG accredited sub-atomic spectrum of massive particles can now be approached for more extensive and thorough analysis on a simple and easily comprehendible basis as a single, continuously linked, composite structure (or composite structural continuum) over many orders of magnitude of systematicly regular distributions of quantally unique masses with a very limited small number of quantized charge characteristics.

ACKNOWLEDGEMENTS---I thank Fred E. Howard, III, for many constructive comments, H. Blevins Howard for assistance with figures, and Cheryl Mack and Christi Rountree of the US Air Force Armament Laboratory Technical Library for patient assistance with related searches of the literature.

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Figure Captions for

SUB-STRUCTURE POWER LAWS OF PARTICLE MASSES --SYSTEMATIC CONSTRAINTS--A NEW CO-ACTIVE LAW --LEPTON-QUARK-HADRON STRUCTURAL CONTINUITY

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Fig. 1 New type of graph of estimated masses of the leptons and quark/anti-quark (LQ) particles (including the neutrinos) as the baseline for a new form of analysis of the cyclicly varied influence of the quantized substructural elements within the *F* factor in calculating the masses of those conventionally elementary (indivisible) particles from the previously reported sub-structural mass/charge power law system in its usual (U) range of application. F varies only over a range factor of 3, slightly less than half an order of magnitude, while the LQ particle masses spread over 10 orders of magnitude, conventionally measured in electron-Volts of mass-equivalent energy. (Use a transparent ruler on the top and bottom mass scales to guide the eye.) Locations for optional mass sites increase under the power law in the displayed line steps with the 6th power of the number of generalized component quanta of mass. Mass also increases with perfect regularity in quantized steps along each steeply sloped, parallel line with F, which is a function of the ratio p/n of the number of charged pairs of those quantized components to the total number of those generalized pairs on that pair number line. The total includes the pairs left in neutral balance with a positive 1/6 charge component and a negative 1/6 component. Those p/n ratios are shown by each particle site. The perfectly regular logarithmic advance with mass along each line and also from widely separated lines to much overlapped lines is clear. Up through the 8 pair line the optional ratio mass sites that are not estimated to be occupied by any known particle are shown as empty steps on the line by small dots without circles around them. Above that, the empty pair number lines with no particle on them are not completed. Above the 16 pair line, empty lines are omitted, though the logarithmicly regular empty sites and lines actually continue, as the short lines at 20/20 indicate. Neutral neutrinos estimated to be occupying a site are shown by small circles, with the two principal ones filled in and named, nu mu and then nu tau, which is the highest upper limit of neutrino mass in the accredited PDG Listings. Sites for the three unit charged leptons, the electron e, the muon mu, and the tau, have larger open circles. The 1/3 and 2/3 charged quark/anti-quark sets are shown by Xs. The conventional quark set names are shown by their first letters. There are in this system at least two estimated mass sites within the PDG empirical mass tolerance for each quark set. Though there are area groupings of known particles by type, the areas are not separated, but overlay each other. Three sites are estimated to be doubly occupied by known particles with the same estimated p/n ratio and mass, but of different particle types. Occupied sites and optional empty sites are intermixed across the graph in particle mass distributions. All the sites are perfectly regular in computed charge and mass sequence under the power law, and known particles appear only on sites in that regular estimated step sequence, but the regularity of specific site occupations by particular particles and types of particles is not readily apparent. Those are the additional regularities of particle mass distributions among the regular sites to be resolved in this paper by the study of numeric relations revealed for the first time by this new graph.

Fig. 2. The first step in resolving the inherent constraint factors in the unambiguous regularity of LQ mass distributions in the U range of the power law system, is that along the line of 0 charged pairs with $F = 0.5$, only neutral neutrinos can occur. Thus only neutrinos may have the uniquely quantized and discrete estimated masses that can be calculated from the power law equation (1) with the point numbers of pairs of components at the base of each pair number line and those *n/p* charge conditions. Similarly, along the lowest *F* line of longer dashes connecting the points with one charged pair out of any total number of pairs, only quark sets with a 1/3 net charge level, whether + or –, can occur and have those unique mass/charge point features. However, on the next higher long-dashed line for two charged pairs, either neutralized neutrinos with 0 net charge or quark sets at 2/3 net charge level can occur and have those unique estimated masses, but only particles meeting those quantized charge conditions in the F factor of the power law can do so. In fact both occur at the 2/4 site with the same mass, but with the alternate net charge levels required and permitted. On the next higher long-dashed line, with 3 charged pairs, only net charge levels of 1 or 1/3 can occur with the family of unique masses at crossings of the pair number lines. Then these conditions alternate cyclicly across the nest of the graph, though the higher *F* areas get too crowded to show them all on a small figure. Thus all sites are at least 50% constrained. Any given charge level observed in LQ particles can occur on only half the sites in this U range of the power law system.

A second important element of regularity in this system is the computed counting of steps in charge/mass build-up. One flyback step must be counted, as shown indicatively by lines of very short dashes for three of the actual 28 cases, all the way across the graph, in moving up in line number from the high F end of one pair number line to the low F end of the next higher pair number line without omitting any steps or lines, whether occupied or shown on the graph or not. At the beginning of the count in the low mass and low *F* range, there is also a counted step from site 0/0 to site 0/1. This computed counting is used in a new equation.

Fig 3 The segment of the graph that contains the quadrilateral figure formed by the four sites occupied by the up and down quark sets at the lowest mass range of the six kinds of quark sets. Two estimated masses within PDG empirical mass limits for each of the two lowest mass quark sets, up and down, are emphasized on their parallel pair number lines, which have the same positive slope with increasing mass, as shown by the dashed vertical reference and arrows. The parallel emphasized lines are then connected to form a quadrilateral (quad) with two top and bottom end lines that cannot be parallel because both the unlike 2/3 and 1/3 net charge levels of the up quark set and the down quark set (respectively) and also the logarithmic spread of net charge level lines would prevent it. Though not quite a parallelogram, the figure (taken with Fig. 2) embodies the basic mass regularity of their mass/charge relations. No other sites can have equivalent features with estimated masses as low, which correlates suggestively with their status as the components of protons, the primary stability feature of atomic nuclei. Such particles could not occur in this power law system except at these vertices.

Fig. 4. A larger segment of the basic graph contains the two quads formed by the six sites occupied by the up, down, and strange quark sets. A nearest equivalent to the emphasized lines of Fig. 3 is drawn between the regular estimated mass sites for the strange quark set. Since these pair lines are well above the beginning of line overlap, the PDG mass range window can and does contain two estimated mass sites on two pair number lines. Thus the set line slope is negative, as shown by the arrow and vertical reference line. The relations of these estimated masses with the up and down set must be well away from a simple repetition regularity, as is disclosed by the distorted shape and spacing of this second quad, crossing the line overlap zone boundary. Yet all six of these estimated masses were substantiated in the prior reference note, by being necessary and sufficient to construct and effectively explain the complex empirical mass structure of two sets of composite PDG hadron (baryon) particles in accordance with the equations of the power law system. The mass distribution regularity that constrains these estimated strange set masses away from the empty sites around them to these specific sites is not immediately apparent from this figure alone, even though the PDG empirical window data appears to fulfil the constraint necessity.

Fig. 5. All five quads formed by the sites occupied by all six kinds of quark sets, up, down, strange, charm, bottom, and top quark sets. The first letters of these names are used to identify the occupied optional sites which correlate with the PDG empirical masses. A systematic pattern of overall mass distribution emerges from adding the emphasized lines for the other three empirical quark sets, charm, bottom, and top, with the additional numbers of sites within or near the windows of their empirical mass uncertainties. The lines form three additional pseudo-quads, near quads in appearance, with their multiple vertices at more sites per line than the two sites per quark set in the first three sets. The third quad-like shape denotes a space of constraint readjustment before a modified set of systematic mass constraints is resumed in the fourth and fifth quads similarly at higher mass to the mass/charge constraints of the first and second quads. The text finds systematicly regular numerical relations consistent with that view. It also points out a need for substantiation tests of the multiple mass sites in each of the three higher mass quark sets similar to those already done for the three lower mass quark sets.

Fig. 6. The emphasized lines of connection between the three sites of unit charged leptons (UCLs) in an overlay of the quads formed by the quark sets. After derivation in the text of a new exponential equation law for estimated mass sites as a function of the sum of power law steps in mass/charge build-up along the pair lines, the resultant mass ratios of the UCLs (the electron, muon, and tau) confirm their mass distribution regularity and contraint, from the interaction of the two equations.

Fig. 7. Differently emphasized lines for each of the tyes of LQ particles from the prior figures. The regularly repeated pattern of a decrease in *F* with mass, followed by an increase with mass to a lower *F* value, is evident in each of the three particle types. (See text for details.) The broadened similarity of the continuous line of the UCL mass distribution V pattern to the V pattern repeated in the heavy dashed lines of the quark set quads, as well as in the heavy dotted principal lines of the neutrinos, implies strongly that similar interactions within the three different types of mass/charge relations constrain the cyclic regularity of mass distributions of all the LQ particles to their very limited number of specific sites. The interaction that defines the V on the graph has the form of a decrease in F (where it is not already at the minimum) with increasing mass, followed by an increase in F with increasing mass in each kind of particle. In the quark sets the cycle is repeated. This cyclicly repeated distribution of masses illustrates the analysis in the text of the relations between quantized mass/charge components in composite LQ structures, with conserved charge, as described by the sub-structural mass/charge power law and the co-active exponential

law, which define and control the computed masses of these conventionally elementary particles, in correlation with the empiricly observed masses.